

The ELF Model

– An operational low flow forecasting system for British Columbia

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BC River Forecast Centre

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The Extrapolating Logarithmic Flow (ELF) Model



Koksilah River, June 18, 2019

1. Why ELF Model – drought in BC
2. What is low flow and characteristics
3. Why not a hydrological model
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7. Data issues and twelve-step and twelve-scenario scheme
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11. Forecasts for regulated stations
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2015 DROUGHT LEVELS AT A GLANCE														
Drought Levels:	1	Normal		2	Dry		3	Very Dry		4	Extremely Dry			
Basins	15-May	28-May	25-Jun	03-Jul	09-Jul	15-Jul	21-Jul	23-Jul	05-Aug	06-Aug	20-Aug	03-Sep	17-Sep	01-Oct
Northwest	2	1	1	1	1	1	1	1	1	1	1	1	1	1
Stikine	2	1	1	1	1	1	1	1	1	1	1	1	1	1
Northeast	1	1	1	1	2	2	2	2	2	2	1	1	1	1
Peace	1	1	1	1	2	2	2	2	2	2	2	1	1	1
East Peace	1	2	1	1	2	2	2	2	2	2	2	1	1	1
Skeena-Nass	1	1	1	1	1	1	1	2	2	2	2	1	1	1
Nechako	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Upper Fraser	1	1	1	1	1	1	1	2	2	2	2	2	1	1
Upper Columbia	1	1	1	1	1	1	1	2	2	2	2	2	1	1
Lower Columbia	1	1	2	2	2	2	2	2	2	3	3	3	2	2
West Kootenay	1	1	2	2	2	2	2	2	2	3	3	3	2	2
East Kootenay	2	1	2	2	2	2	2	2	2	3	3	3	1	1
Middle Fraser	2	1	1	1	1	1	1	2	2	3	3	3	2	1
North Thompson	1	1	1	1	2	2	2	2	3	3	4	4	2	1
South Thompson	1	1	2	2	3	3	3	4	4	4	4	4	3	2
Okanagan-Kettle	2	2	2	2	3	3	3	3	4	4	4	4	3	3
Kettle (separated July 23)								4	4	4	4	4	3	3
Nicola	2	2	2	2	3	3	4	4	4	4	4	4	3	2
Similkameen	2	2	2	2	3	3	3	4	4	4	4	4	3	2
Skagit	1	1	1	1	3	3	3	4	4	4	4	3	2	1
Lower Fraser	2	2	3	3	3	4	4	4	4	4	4	3	2	1
South Coast	2	2	3	3	3	4	4	4	4	4	4	3	2	1
Vancouver Island	2	3	3	4	4	4	4	4	4	4	4	3	2	1
Haida Gwaii	1	3	3	3	3	3	3	3	3	3	2	1	1	1
Central Coast	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Prepared By: Water Management Branch, Ministry of Forests, Lands and Natural Resource Operations
Last Update: October 2, 2015

2021 DROUGHT LEVELS AT A GLANCE


Drought Levels:	0	1	2	3	4	5																
BASINS	26-May	09-Jun	23-Jun	07-Jul	14-Jul	21-Jul	28-Jul	04-Aug	11-Aug	18-Aug	20-Aug	25-Aug	01-Sep	08-Sep	15-Sep	17-Sep	22-Sep	29-Sep	06-Oct	20-Oct	28-Oct	01-Nov
Fort Nelson	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
East Peace	0	0	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
North Peace	0	0	0	0	2	2	2	2	2	2	2	0	0	0	0	0	1	1	0	0	0	0
South Peace	0	0	1	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
Northwest	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Stikine	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Skeena-Nass	0	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
Bulkley-Lakes	0	0	0	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
Finlay	0	0	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0	0	0	0	0
Parsnip	0	0	0	0	0	1	1	1	1	1	1	1	1	2	1	1	1	0	0	0	0	0
Upper Fraser West	0	0	0	2	2	2	1	1	1	1	1	1	1	2	2	2	1	1	1	0	0	0
Upper Fraser East	0	0	0	1	2	2	2	2	2	1	1	1	2	2	1	1	0	0	0	0	0	0
Upper Columbia	0	0	0	0	1	1	2	2	2	1	1	1	2	2	2	2	1	0	0	0	0	0
Lower Columbia	1	1	1	2	3	4	4	4	4	4	4	4	4	4	4	4	4	3	2	2	1	1
West Kootenay	1	1	1	2	3	4	4	4	4	4	4	4	4	4	4	4	4	3	2	2	1	1
East Kootenay	1	1	1	1	2	2	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2
Kettle	1	2	2	3	4	4	4	5	5	5	5	5	5	5	5	5	5	4	3	3	2	2
Middle Fraser	1	1	1	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	0	0	0	0
North Thompson	0	0	0	1	3	3	4	4	4	4	4	4	4	4	4	4	3	3	3	0	0	0
South Thompson	1	1	1	2	3	3	4	4	4	4	4	4	4	4	4	4	4	4	3	1	1	1
-Salmon River	1	2	2	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	3	3	3
Nicola	1	1	2	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	3	1	1	2
-Coldwater River	1	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	3	1	1	1
Okanagan	1	1	1	3	3	3	3	3	4	4	4	4	4	4	4	4	3	3	3	2	2	2
Similkameen	0	0	1	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1	1
Skagit	1	1	1	1	2	3	3	3	3	3	3	3	3	3	3	3	2	1	0	0	0	0
Lower Fraser	1	1	1	1	2	3	4	4	4	4	4	4	4	4	3	3	2	1	0	0	0	0
South Coast	1	1	1	1	3	3	4	4	4	4	4	4	4	4	3	3	2	1	0	0	0	0
Central Coast	0	0	0	0	0	0	0	0	0	1	1	1	2	2	2	2	0	0	0	0	0	0
West Vancouver Island	1	1	1	3	3	3	4	4	4	4	5	5	5	5	5	4	2	0	0	0	0	0
East Vancouver Island	2	2	2	4	4	4	4	4	4	4	5	5	5	5	5	4	2	1	0	0	0	0
Haida Gwaii	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Prepared By: Water Management Branch - Ministry of Forests, Lands, Natural Resource Operations and Rural Development


2023 DROUGHT LEVELS AT A GLANCE

Drought Levels:	0	1	2	3	4	5																			
BASINS	01-Jun	08-Jun	15-Jun	22-Jun	29-Jun	06-Jul	13-Jul	20-Jul	27-Jul	03-Aug	10-Aug	17-Aug	24-Aug	31-Aug	07-Sep	14-Sep	21-Sep	28-Sep	05-Oct	12-Oct	19-Oct	26-Oct	02-Nov		
Fort Nelson	2	3	4	4	4	4	5	5	5	5	5	5													
East Peace	3	3	4	4	4	4	4	4	4	4	4	4													
North Peace	1	2	3	3	4	4	4	4	4	4	4	4													
South Peace	1	2	2	2	3	4	4	5	5	5	5	5													
Northwest	0	0	0	0	1	1	1	1	1	1	1	1													
Stikine	0	0	0	0	1	2	2	2	2	2	2	2													
Skeena-Nass	0	1	1	2	3	3	3	3	3	3	3	3													
Bulkley-Lakes	1	2	3	3	4	4	5	5	5	5	5	5													
Finlay	1	2	2	4	4	4	4	5	5	5	5	5													
Parsnip	1	2	2	3	4	4	4	5	5	5	5	5													
Upper Fraser West	1	2	2	3	4	4	4	5	5	5	5	5													
Upper Fraser East	1	2	2	2	4	4	4	4	4	4	4	4													
Upper Columbia	2	2	3	3	3	3	3	3	4	4	4	4													
Lower Columbia	1	2	3	3	3	3	4	4	4	4	4	4													
West Kootenay	1	2	3	3	3	3	4	4	4	4	4	4													
East Kootenay	2	2	3	3	3	3	4	4	4	4	4	4													
Kettle	0	2	3	3	3	3	3	3	3	4	4	4													
Middle Fraser	1	2	2	2	3	3	4	3	3	3	3	3													
Lower Thompson	0	2	2	2	2	3	3	3	3	4	4	5													
North Thompson	0	2	2	2	3	4	4	4	4	5	5	5													
South Thompson	0	2	2	2	3	4	4	4	4	5	5	5													
-Salmon River	0	3	3	3	3	4	4	4	5	5	5	5													
Nicola	1	2	2	2	2	3	3	3	3	4	4	5													
-Coldwater River	2	3	3	3	3	4	4	4	4	4	4	5													
Okanagan	0	2	2	2	2	3	3	3	3	4	4	5													
Similkameen	2	3	3	3	3	3	3	3	4	4	4	5													
Central Pacific Range	1	2	2	2	3	3	3	3	3	3	3	3													
Eastern Pacific Range	2	3	3	3	3	3	3	3	3	4	4	5													
Sunshine Coast	2	3	3	3	4	4	4	4	4	4	4	5													
Lower Mainland	2	3	3	3	4	4	4	4	4	4	4	5													
Central Coast	1	2	2	2	2	2	3	3	3	4	4	4													
West Vancouver Island	2	3	3	3	4	4	5	5	5	5	5	5													
East Vancouver Island	2	3	3	3	4	4	5	5	5	5	5	5													
Haida Gwaii	0	1	2	3	3	3	4	4	4	3	3	2													

Appendix 4: Provincial and Federal Agency Drought Responsibilities



British Columbia Drought and Water Scarcity Response Plan



Updated April 2023

Prepared by the Ministry of Water, Land and Resource Stewardship
on behalf of the Inter-Agency Drought Working Group

Ministry of Forests, (FOR)	<ul style="list-style-type: none"> • Lead provincial agency for drought coordination and response • Administers the <i>Water Sustainability Act</i> • Operates the River Forecast Centre; collects and interprets snow, meteorological and stream flow data to provide warnings and forecasts of stream and lake runoff conditions • Monitors ambient water quality and groundwater levels • Protects and restores fish habitat and aquatic ecosystems
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The RFC collects and interprets snow, meteorological and stream flow data to provide warnings and forecasts of stream and lake runoff conditions around the province.

Appendix 7: Additional Resources

All emergency situations that affect the health and safety of the public should be reported to EMBC at 1-800-663-3456.

Provincial Government Resources

- [General Drought Information B.C. webpage](#): Links to low provincial government drought information including stream flow advisories, handbooks, fact sheets, and more.
- [River Forecast Centre \(RFC\)](#): The RFC collects and interprets snow, meteorological and stream flow data to provide warnings and forecasts of stream and lake runoff conditions around the province.

Flood/High flow forecasting

WANRS
Model for
Fraser/
Skeena
(1976)

COFFEE
Model



RAVEN
Model for
Okanagan
Lake



FRÔUT
Model for
Fraser

The REC collects and interprets snow, meteorological and stream flow data to provide warnings and forecasts of stream and lake runoff conditions around the province.

ELF



What is low flow?

Low flow is the "flow of water in a stream during prolonged dry weather," according to the World Meteorological Organization. Many states use design flow statistics such as the 7Q10 (the lowest 7-day average flow that occurs on average once every 10 years) to define low flow for setting permit discharge limits.

Do low flows occur at the same time each year?

Most streams will illustrate annual variation that can be explained by seasonal changes in snowmelt, rainfall and other factors. For many areas in the country, the lowest flows often occur near the end of the summer or beginning of fall. However, each stream is different and any particular year can be an anomaly in terms of if and when low flows occur. The magnitude and duration of low flows can vary significantly from year to year.

Low flows almost unpredictable!

The screenshot shows the EPA website page for 'Definition and Characteristics of Low Flows'. The page includes a search bar, a navigation menu, and a list of links under the heading 'Low Flow and Droughts: Definitions and Characteristics'. Two yellow arrows point from the text boxes to the links 'What is low flow?' and 'Do low flows occur at the same time each year?'.

Definition and Characteristics of Low Flows

This page provides background information on the definition and characteristics of low flows, the relationship between low flows and aquatic life criteria and design flows.

Low Flow and Droughts: Definitions and Characteristics

- [What is low flow?](#)
- [What is a drought?](#)
- [What is the difference between low flow and drought?](#)
- [What factors affect flow?](#)
- [Do low flows occur at the same time each year?](#)
- [Why does a low flow year begin in April while a water year begins in October?](#)

<https://www.epa.gov/ceam/definition-and-characteristics-low-flows>

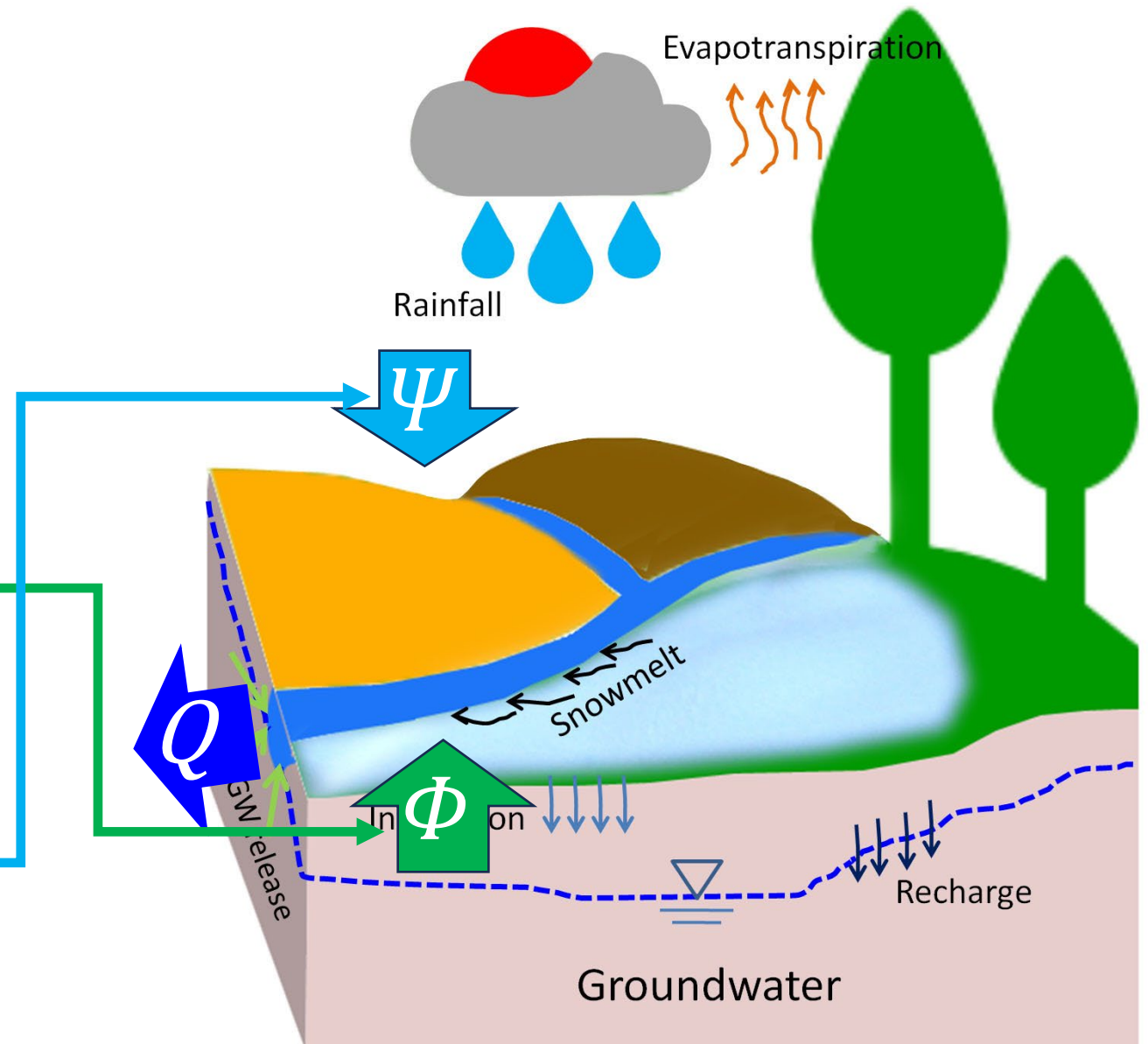
Q - outflow from the watershed:

$$Q = \left(-\frac{dS}{dt} \right) + (R + M - E)A$$

$$Q = \Phi + \Psi$$

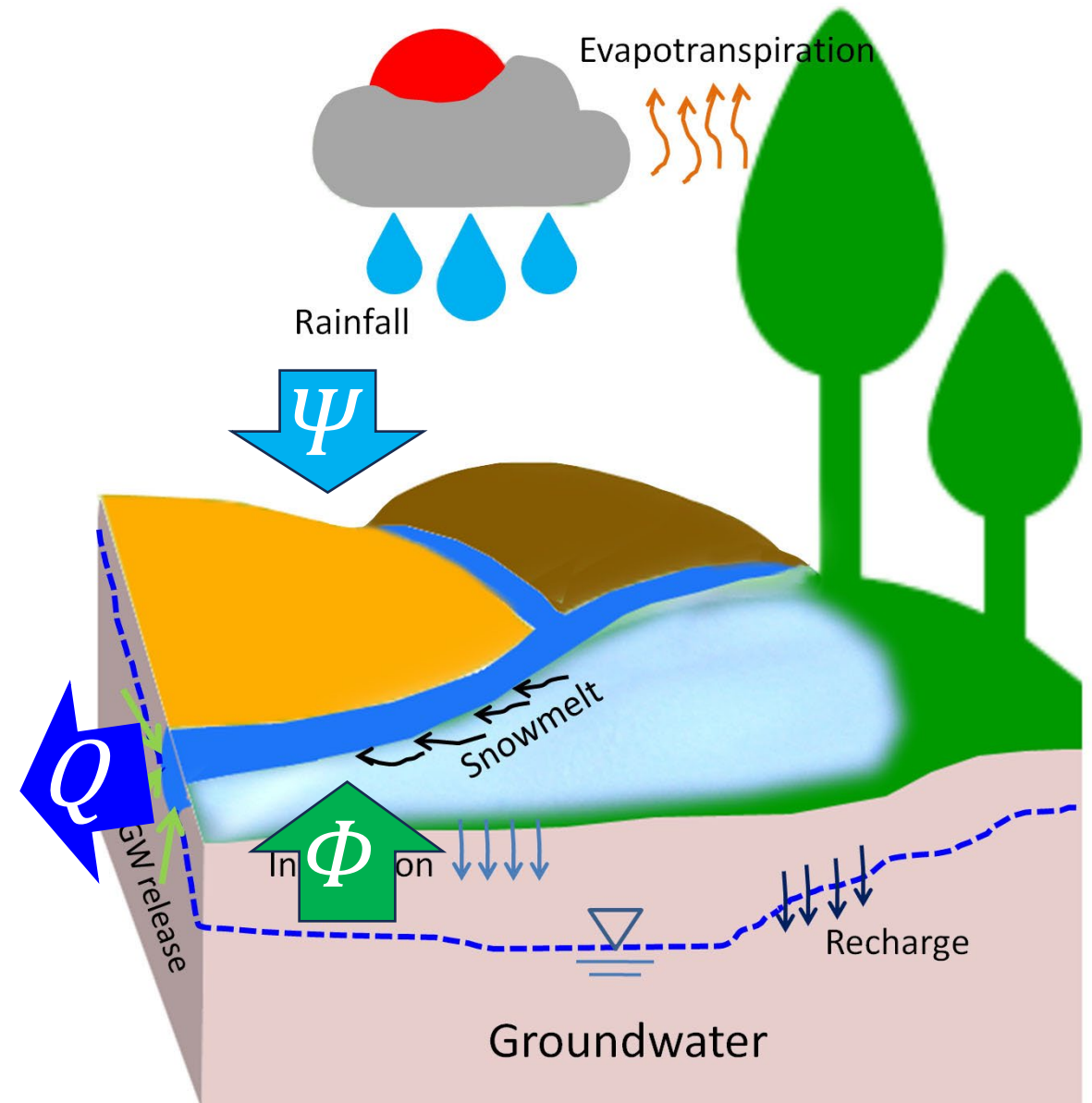
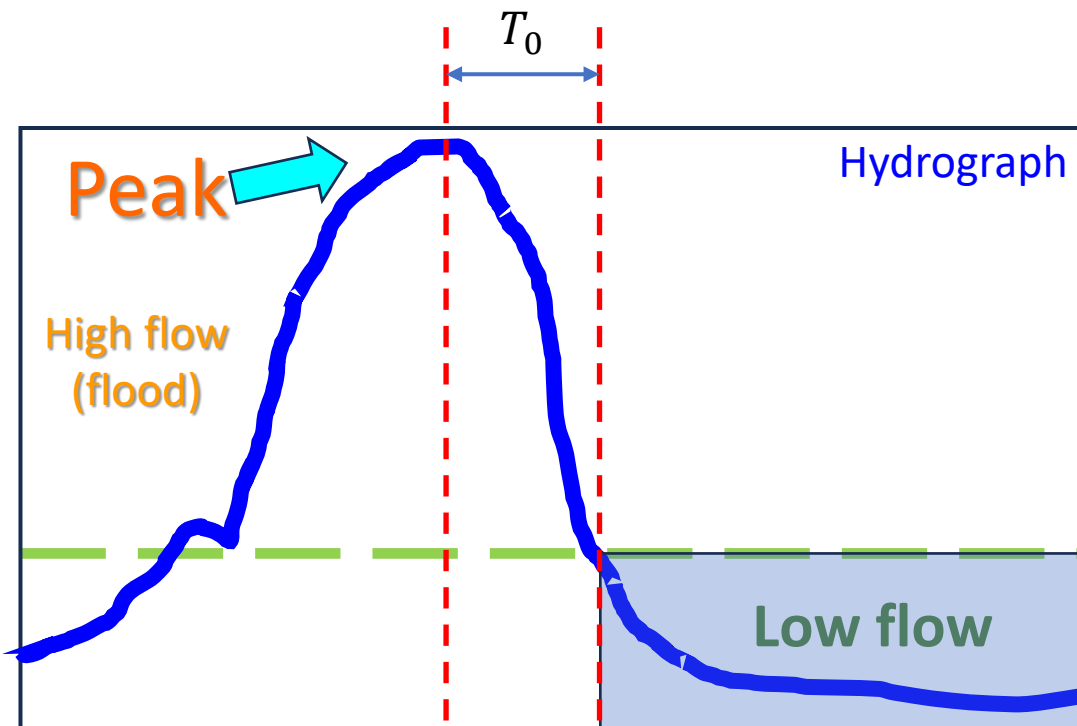
$\Phi = -ds/dt$, the water release rate from the watershed liquid water storage.

$\Psi = (R + M - E)A$, the net meteorological liquid water input rate to the watershed.



Definition: The low flow Q_L is the outflow from a watershed that has been continuously decreasing from the most recent high peak for a period (T_0):

$$\begin{cases} \Delta Q = \Delta(\Phi + \Psi) < 0 \\ Q_L = Q_t, \quad \text{when } t \geq T_0 \end{cases}$$



STATION ID	STATION NAME	A (km ²)	SQRT(A) (km)	MAD (m ³ /s)	½ MAD (m ³ /s)	T ₀ (days)	WTS
08HB086	TOFINO CREEK NEAR THE MOUTH	38.6	6.2	6.8	3.4	3	COASTAL
08GB013	CLOWHOM RIVER NEAR CLOWHOM LAKE	147	12.1	15.4	7.7	5	COASTAL
08MH147	STAVE RIVER ABOVE STAVE LAKE	290	17.0	34.5	17.2	5	COASTAL
08GF007	WAKEMAN RIVER BELOW ATWAYKELLESSE RIVER	698	26.4	78.1	39.1	5	COASTAL
08GA071	ELAHO RIVER NEAR THE MOUTH	1200	34.6	105.0	52.5	6	COASTAL
08GE002	KLINAKLINI RIVER EAST CHANNEL (MAIN) NEAR THE MOUTH	5780	76.0	299.9	149.9	12	COASTAL
08CG001	ISKUT RIVER BELOW JOHNSON RIVER	9500	97.5	465.0	232.5	33	COASTAL
08DB001	NASS RIVER ABOVE SHUMAL CREEK	18400	135.6	806.1	403.1	18	COASTAL
08CE001	STIKINE RIVER AT TELEGRAPH CREEK	29000	170.3	421.3	210.6	39	COASTAL
08EF001	SKEENA RIVER AT USK	42300	205.7	911.6	455.8	34	COASTAL
08NJ026	DUHAMEL CREEK ABOVE DIVERSIONS	52.9	7.3	1.5	0.8	32	INTERIOR
08NG077	ST. MARY RIVER BELOW MORRIS CREEK	208	14.4	7.1	3.6	29	INTERIOR
08NF001	KOOTENAY RIVER AT KOOTENAY CROSSING	416	20.4	4.9	2.4	36	INTERIOR
08NG002	BULL RIVER NEAR WARDNER	1520	39.0	32.5	16.3	40	INTERIOR
08NN026	KETTLE RIVER NEAR WESTBRIDGE	2140	46.3	28.0	14	26	INTERIOR
08NL038	SIMILKAMEEN RIVER NEAR HEDLEY	5580	74.7	48.6	24.3	30	INTERIOR
08NG065	KOOTENAY RIVER AT FORT STEELE	11500	107.2	172.5	86.2	62	INTERIOR
08LF051	THOMPSON RIVER NEAR SPENCES BRIDGE	55400	235.4	778.1	389	79	INTERIOR
08MC018	FRASER RIVER NEAR MARGUERITE	114000	337.6	1456.9	728.5	78	INTERIOR
08MF005	FRASER RIVER AT HOPE	217000	465.8	2720.4	1360.2	100	INTERIOR

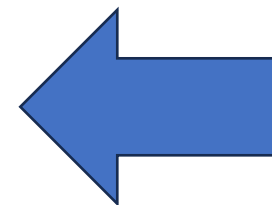
T_0 - receding period

“Recent high peak”

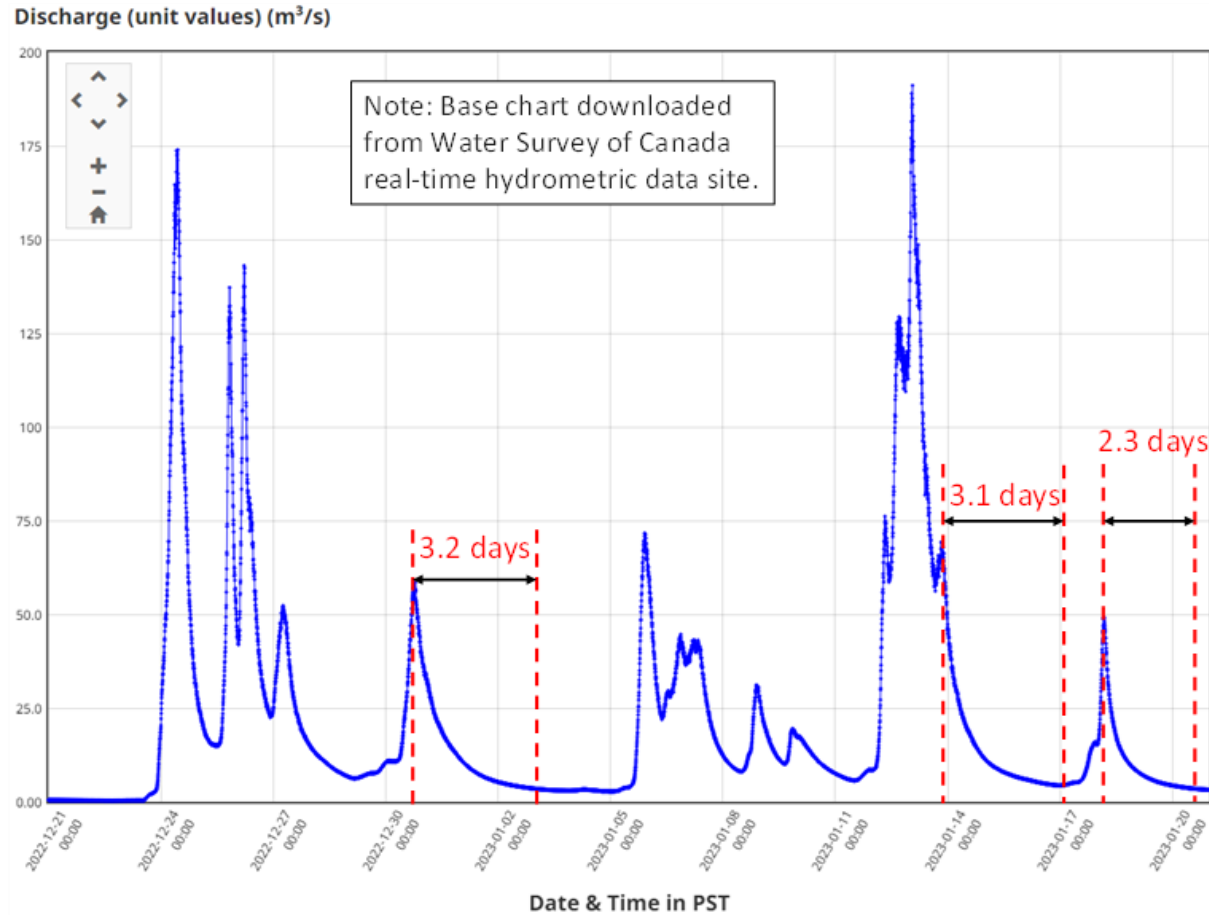
$\geq 2 \times \text{MAD}$ (mean annual discharge)

“For a period”

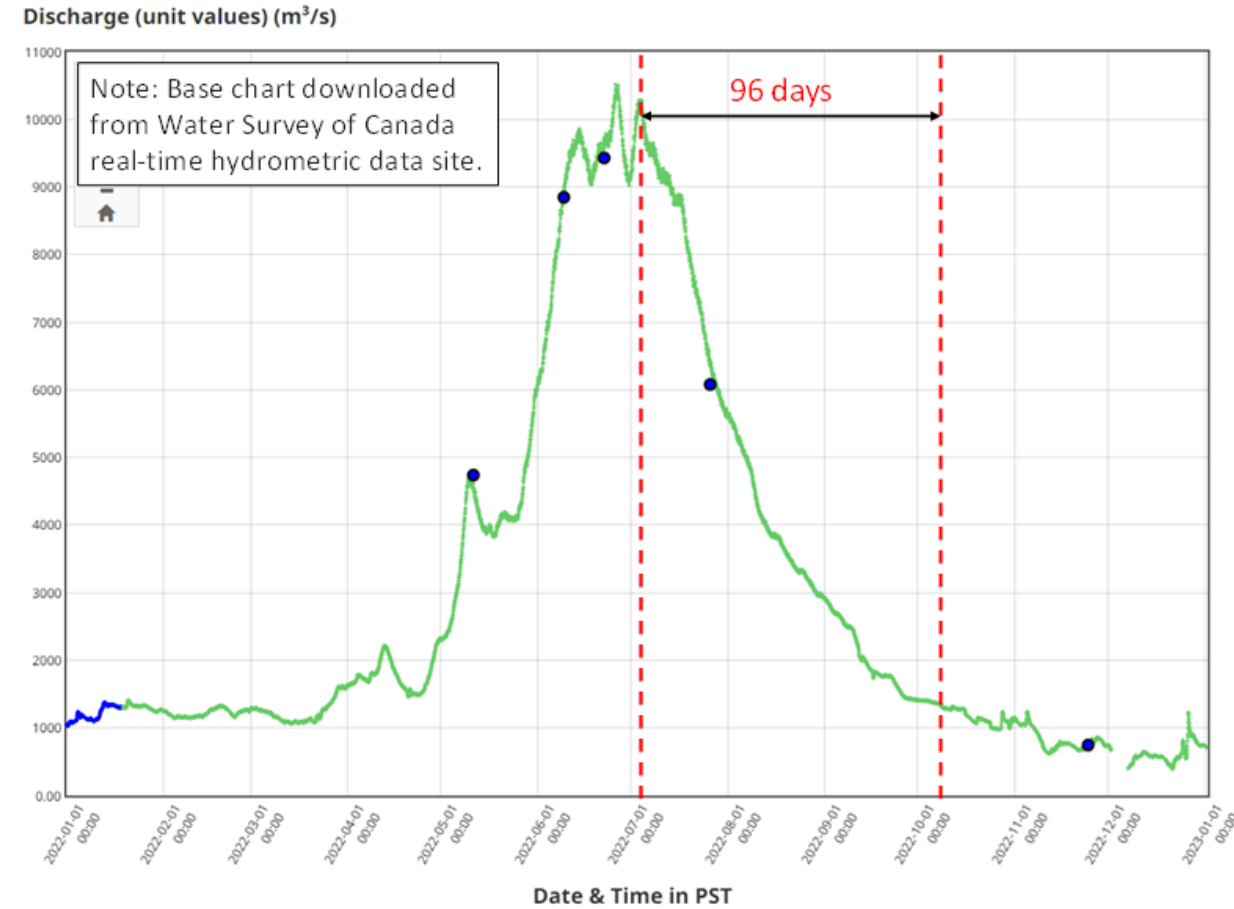
= time from the “recent high peak” until the streamflow reaches $1/2 \text{ MAD}$.



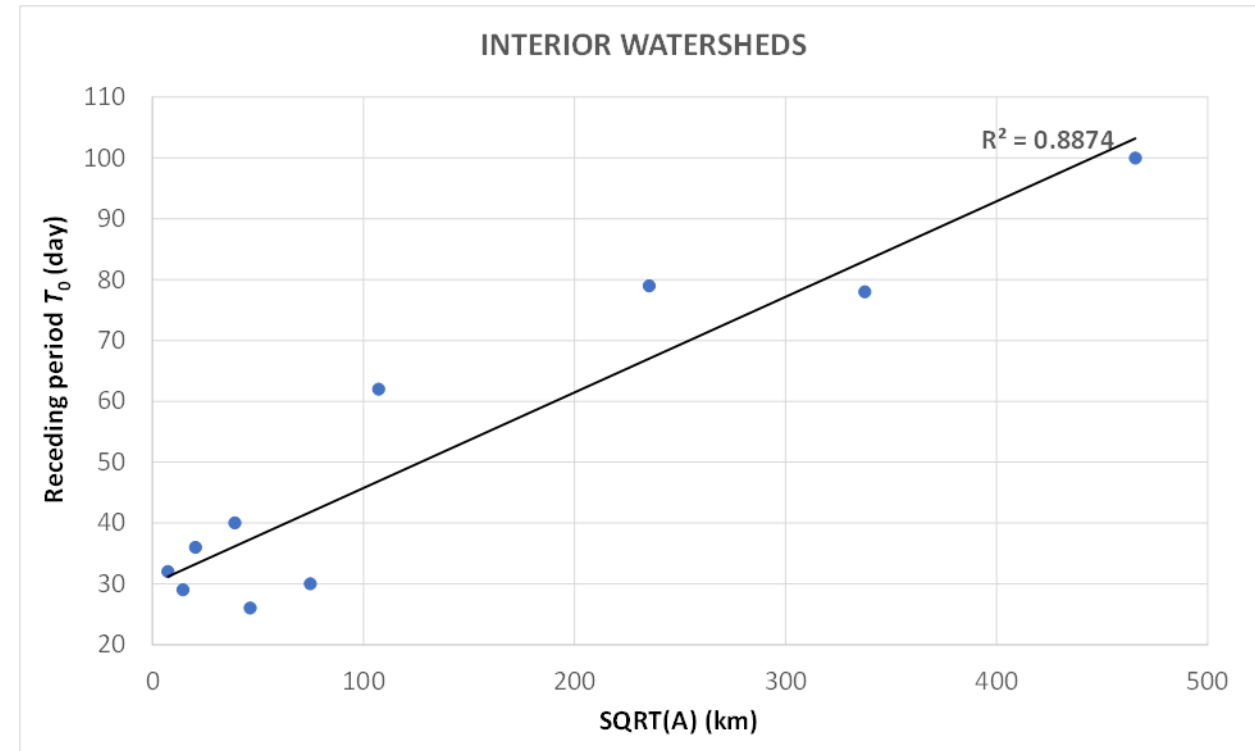
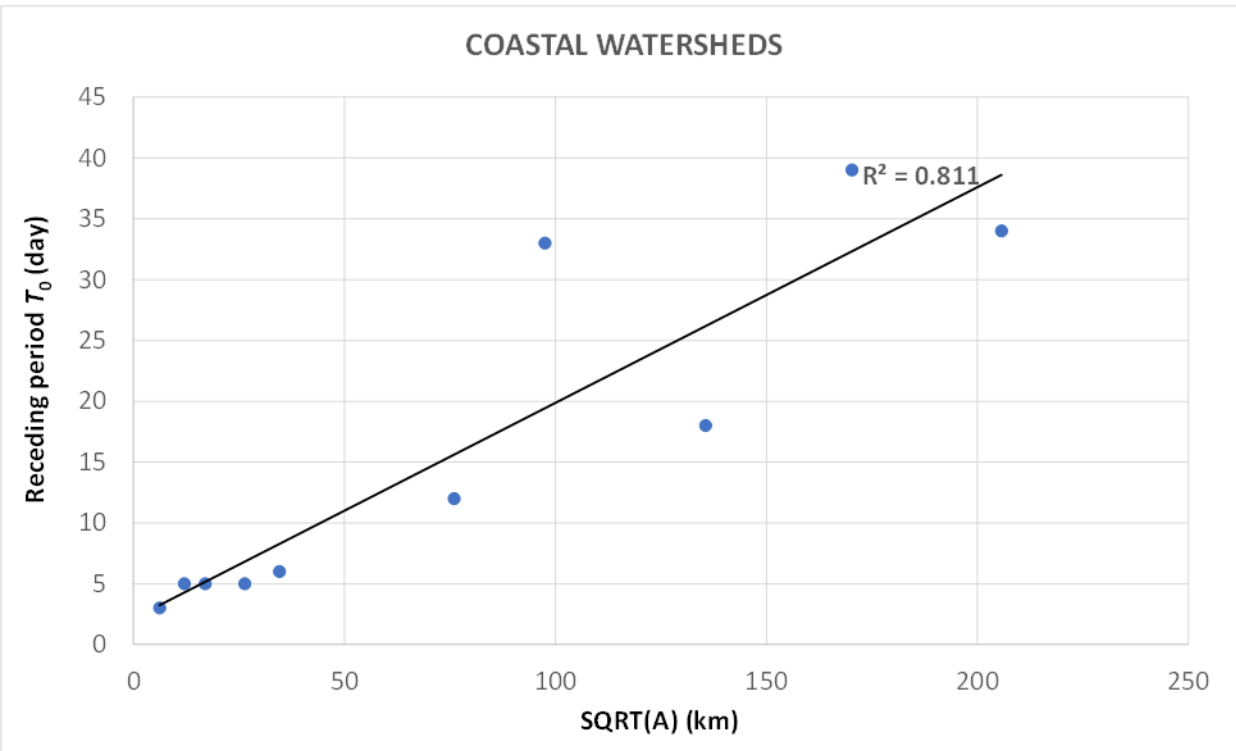
Recent 10 years of data



(a) TOFINO CREEK NEAR THE MOUTH (08HB086): $T_0 = 2.3$ to 3.2 days for Dec. 21, 2022 – Jan. 20, 2023



(b) FRASER RIVER AT HOPE (08MF005): $T_0 = 96$ days for 2022



2016-Jan-31 02:00:00.013 PM
Sproat River, January 31, 2016



Characteristics of low flow:

- The streamflow is **decreasing**.
- The sum of the rate of release from the watershed liquid water storage plus the rate of net meteorological liquid water input is **decreasing**.
- The net meteorological liquid water input is insufficient to replenish the liquid water storage in the watershed.
- The watershed liquid water storage is **decreasing**.

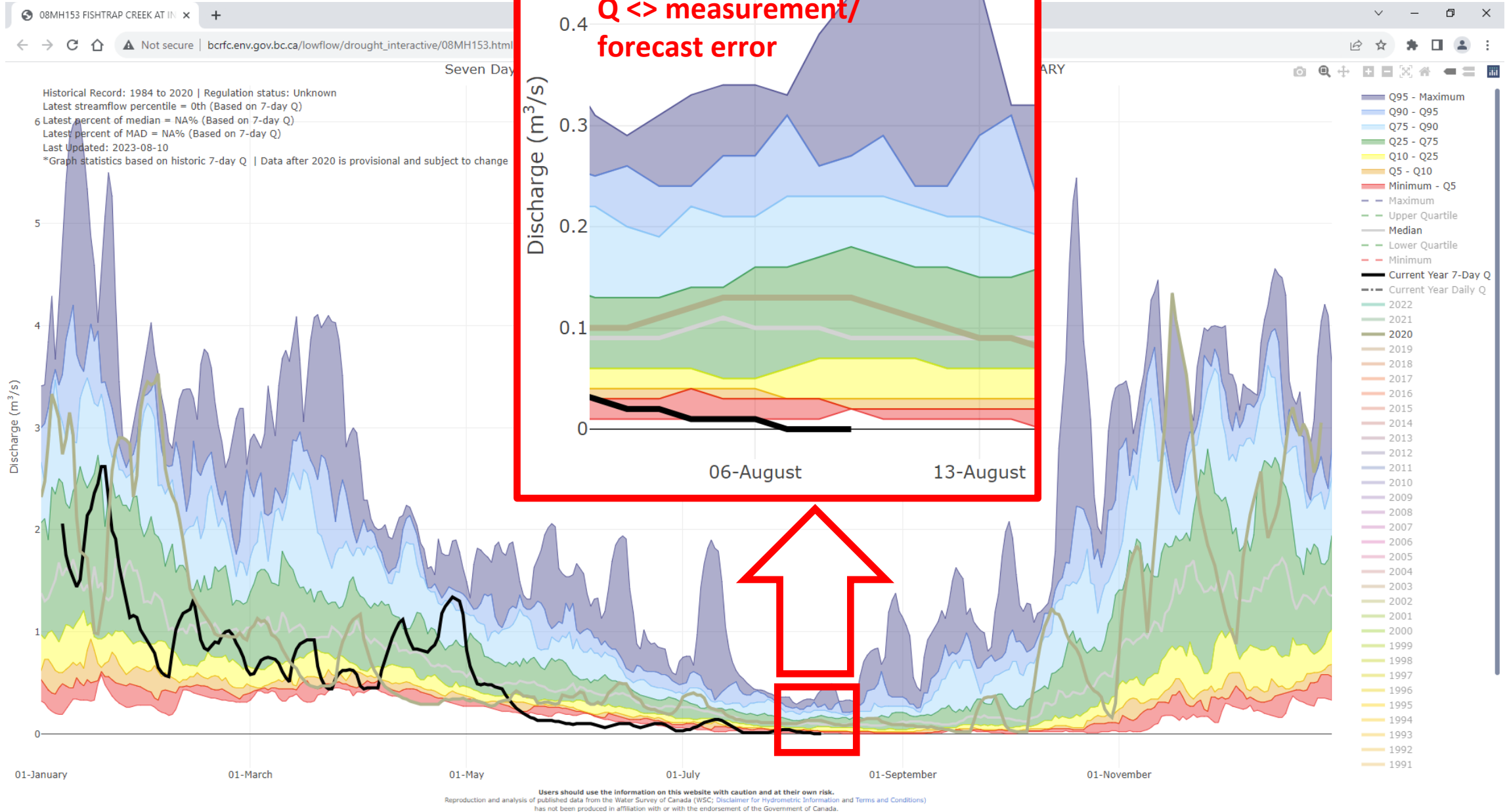
Sproat River Streaming Camera 2023-Aug-22 15:45

Sproat River, August 22, 2023



<http://www.pacfish.ca/wcviweather/Content%20Pages/Sproat/CameraStream3.aspx>

3. Why not a hydrological model



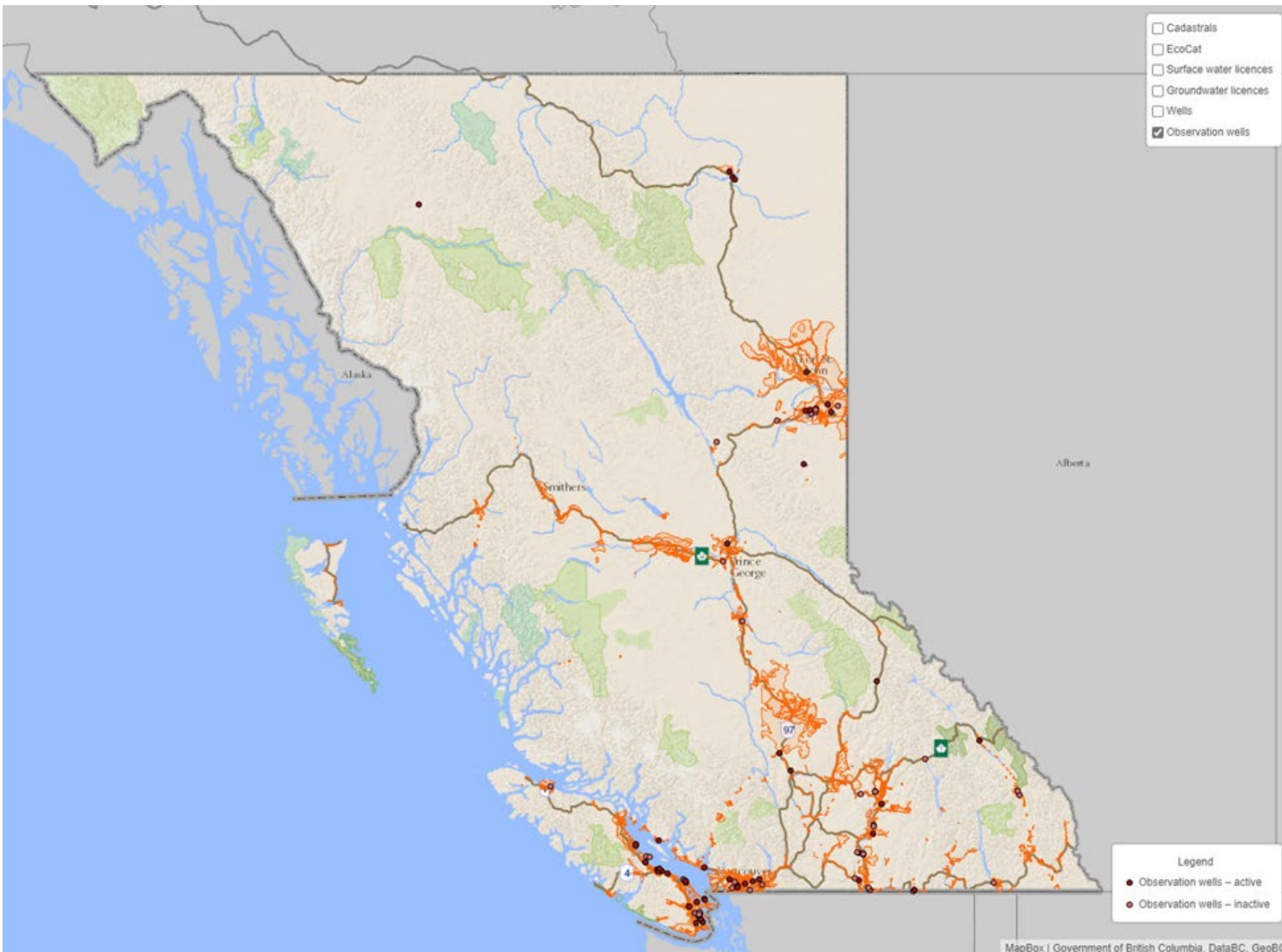
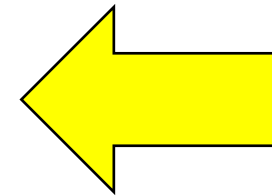
Sparsity of BC observation wells (dark brown dots, 340) and mapped aquifers (brown lines and filled patches) - BC Groundwater Wells and Aquifers website
https://apps.nrs.gov.bc.ca/gwells/aquifers?map_centre=53.810684,-124.817461

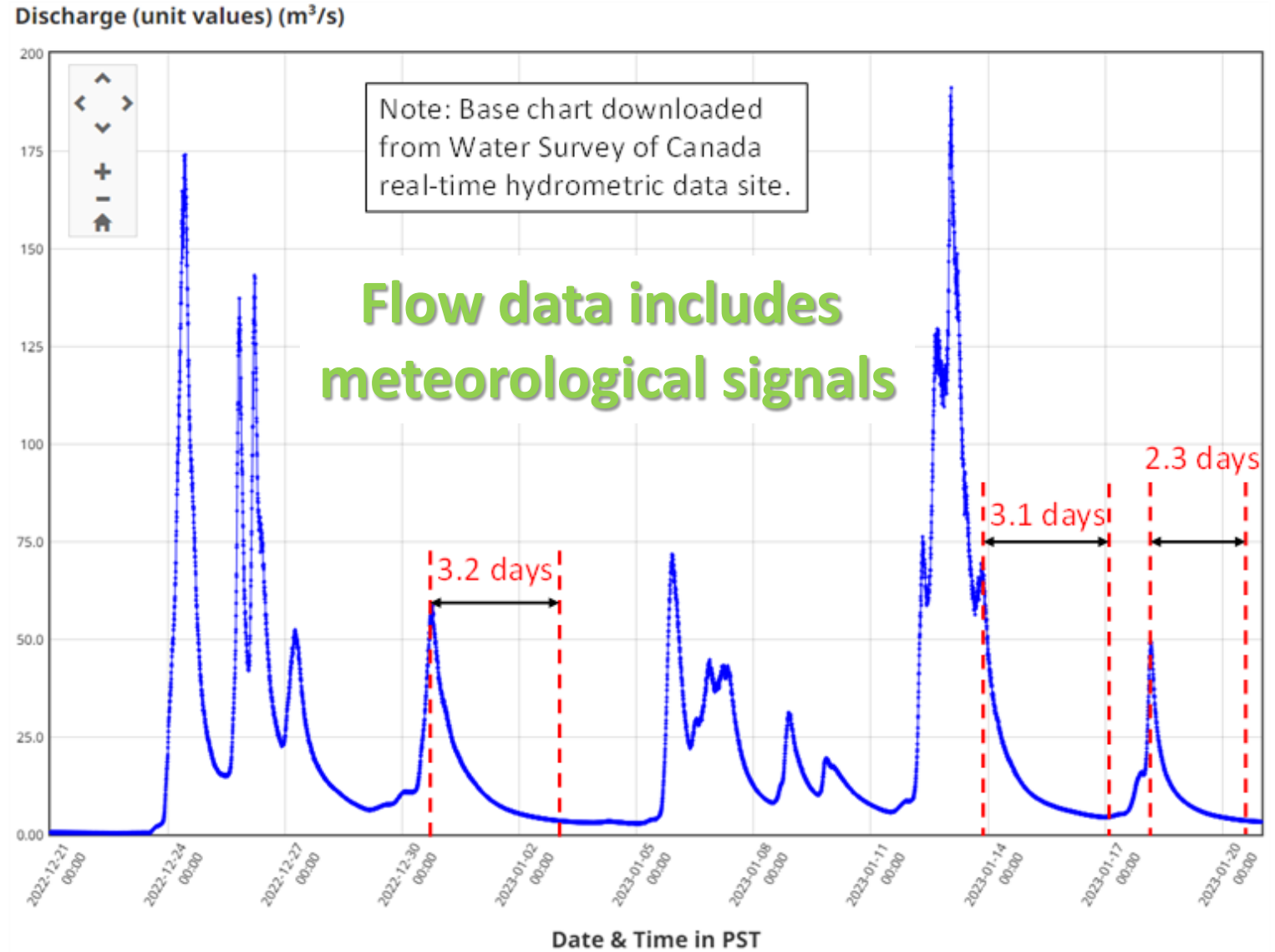
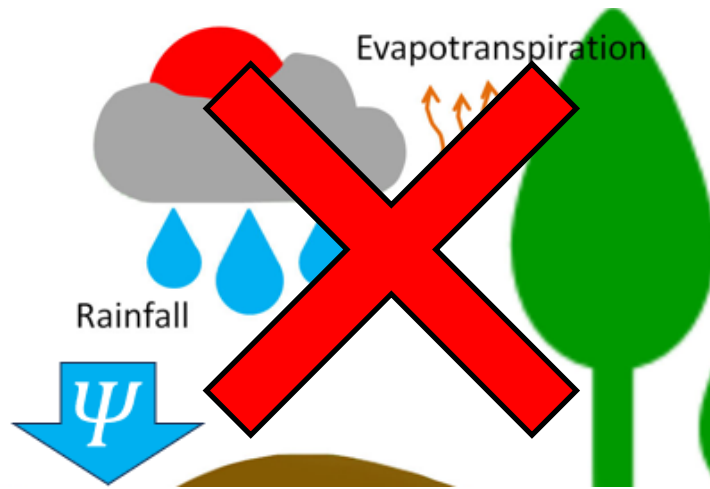
3. Why not a hydrological model

Low flow = baseflow

Conceptual/ Lumped-sum Models:
Baseflow = constant

Physically based distributed Models:
Baseflow = Groundwater release

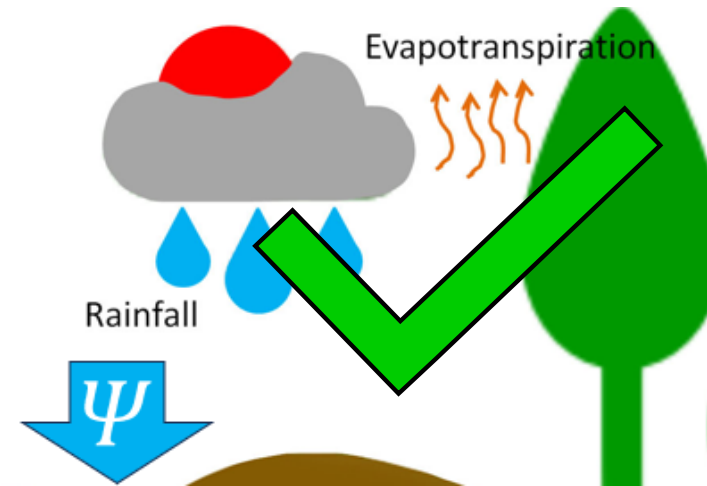
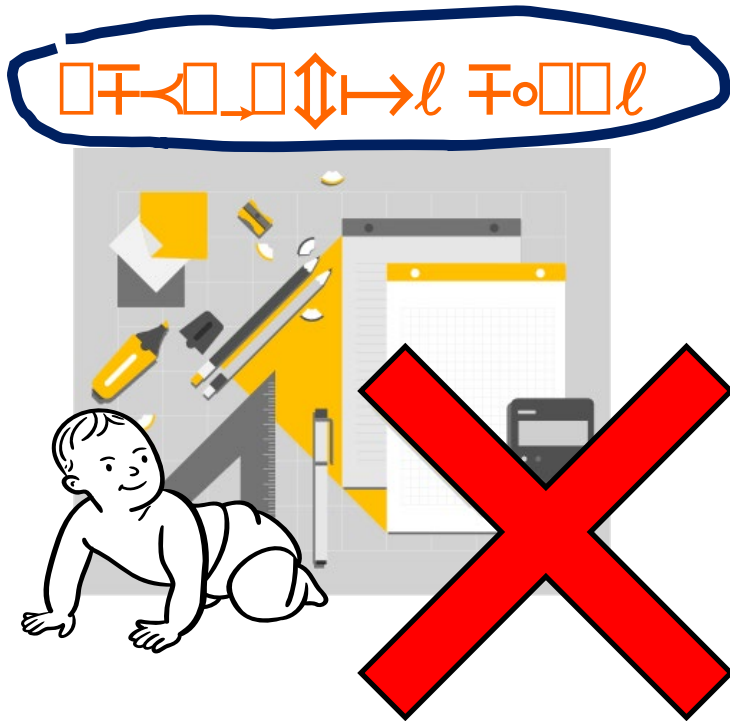




1) The sum of the water release rate from the watershed liquid water storage plus the net meteorological liquid water input rate to the streamflow is a function of time and parameters.

$$Q = \Phi + \Psi = f(p_i, t)$$

2) The function parameters p_i remain constant for a certain period.



$$Q(t) = Q_0 e^{-\alpha t}$$

$$\ln(Q) = -\alpha t + \ln(Q_0)$$

The Extrapolating Logarithmic Flow (ELF) Model

$$\frac{d}{dt} [\ln(Q)] = \frac{d}{dt} [-\alpha t + \ln(Q_0)]$$

$$\frac{1}{Q} \frac{dQ}{dt} = -\alpha$$

$$\frac{dQ}{dt} = -\alpha Q$$

$$\frac{d^2 Q}{dt^2} = \alpha^2 Q$$



- The first characteristic of low flow is extended as,
- The streamflow is **decreasing**, and the decreasing rate of the streamflow becomes **smaller and smaller with time (decreasing)**.

$$y = ax + b$$

$$\begin{cases} y = \ln(Q) \\ x = t \\ a = -\alpha \\ b = \ln(Q_0) \end{cases}$$

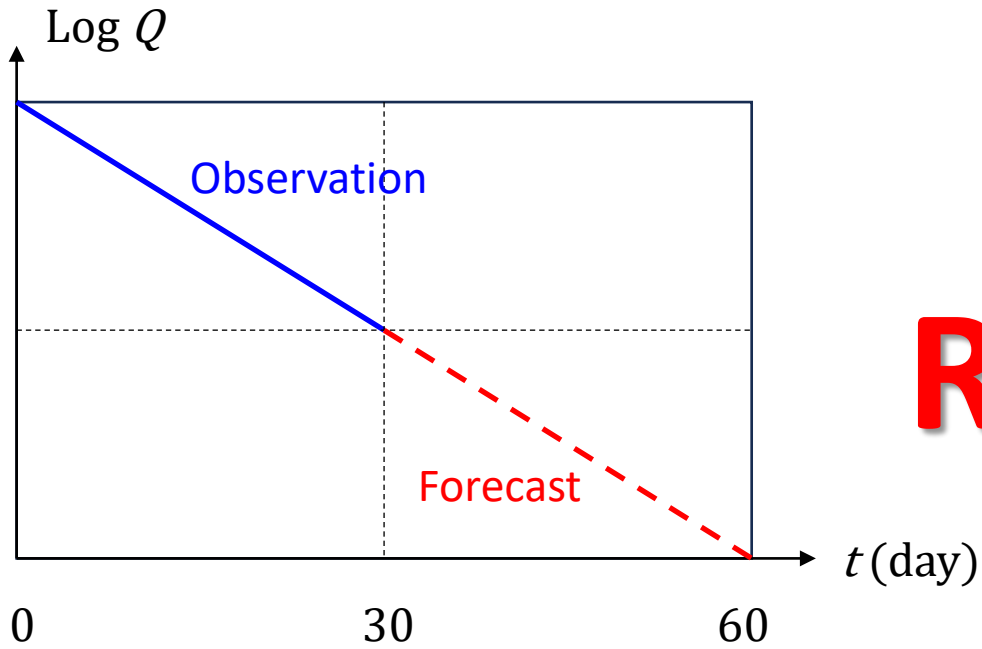
2 unknow: a, b
2 data/equations
Supper easy!!!

If 30 data, overdetermined: $f = y = ax + b$

$$\begin{cases} R^2 = \sum_{i=1}^n [y_i - f(x_i, a_1, a_2, \dots, a_j, \dots, a_m)]^2 \\ \frac{\partial(R^2)}{\partial a_i} = 0, i = 1 \text{ to } n \end{cases}$$

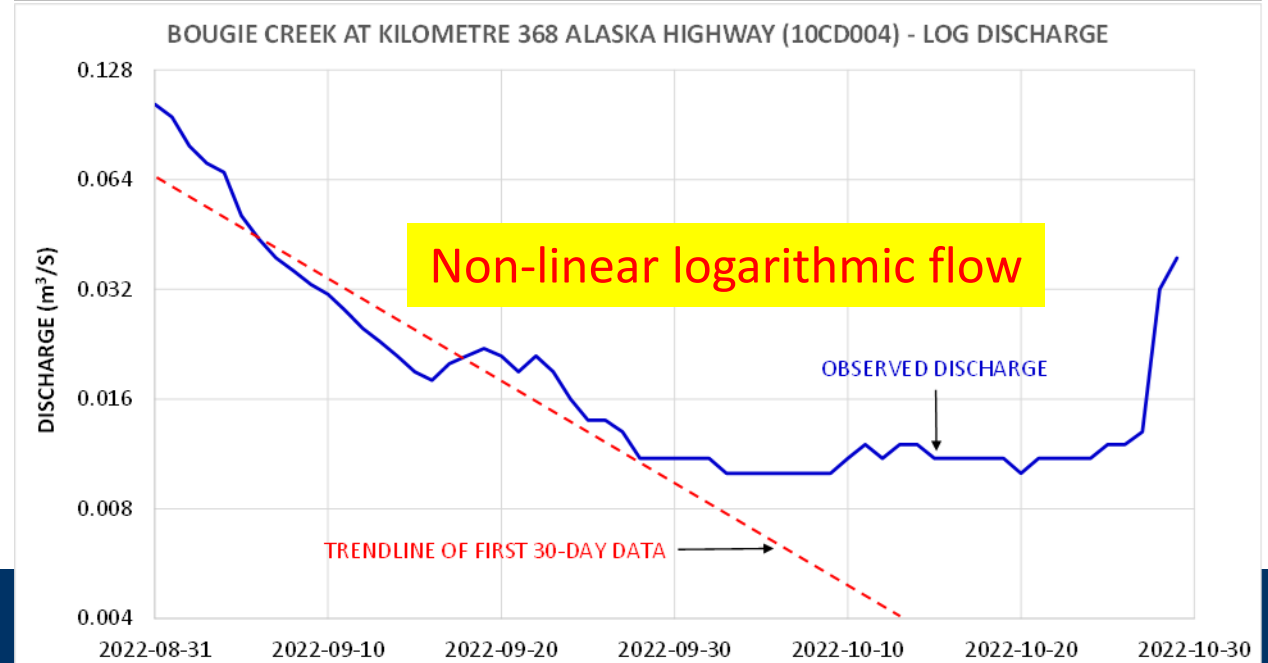
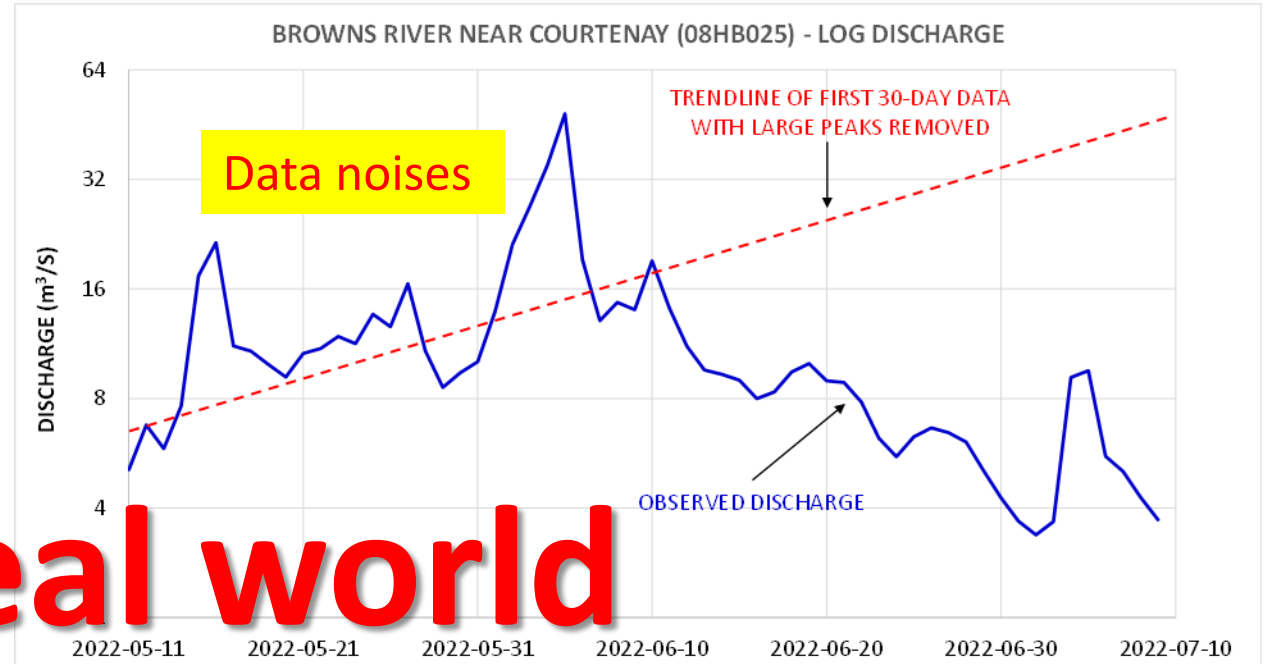
$$\begin{cases} a = \frac{\sum_i^n (x_i y_i) - n \bar{x} \bar{y}}{\sum_i^n x_i^2 - n \bar{x}^2} \\ b = \frac{\bar{y} \sum_i^n x_i^2 - \bar{x} \sum_i^n (x_i y_i)}{\sum_i^n x_i^2 - n \bar{x}^2} \end{cases} \quad \begin{cases} \bar{x} = \frac{1}{n} \sum_{n=1}^n x_i \\ \bar{y} = \frac{1}{n} \sum_{n=1}^n y_i \end{cases}$$





Theory

Real world



ELF Model uses 30-day daily flows as input to produce 30-day forecasts

Step 1. prepare data:

$$\begin{cases} Q_i = Q_{obsi} > 0 \\ H_i = H_{obsi} - H_{min} > 0 \end{cases}$$

Step 2. Calculate the logarithms for observed discharges/water levels

Step 3. Calculate 5-day moving averages:

$$(\overline{\log Q})_j = \frac{1}{5} \sum_{i=d}^{d+4} \log Q_i$$

Step 4. Calculate increment of $(\overline{\log Q})_j$:

$$(\Delta \overline{\log Q})_l = (\overline{\log Q})_{j+1} - (\overline{\log Q})_j$$

$$\Delta \overline{\log Q} \approx d(\overline{\log Q}) \Delta t \approx dy = a \text{ Eq (31)}$$

Or

$$\underline{\Delta \log Q} \approx \underline{d(\log Q)} \Delta t \approx \underline{dy} = \underline{ax + b} \text{ Eq (32)}$$

Step 5. Calculating the increment of $(\Delta \overline{\log Q})_l$:

$$\Delta \Delta \overline{\log Q} \approx d(d(\overline{\log Q})) \Delta t \approx d(dy) = da = 0$$

Or

$$\Delta \Delta \overline{\log Q} \approx d(d(\log Q)) \Delta t \approx d(dy)$$

$$= d(ax + b) = a$$

Step 6. Scenario 1 – Fit 25 points of $\Delta \overline{\log Q}$ in Eq (32): $\underline{\Delta \log Q} \approx \underline{d(\log Q)} \Delta t \approx \underline{dy} = \underline{ax + b}$

Step 7. Calculating the mean of $(\Delta\Delta \overline{\log Q})_n = 0$ or non-0 constant

Step 8. *Scenario 2 to 8* – Eliminate data points of $(\Delta \overline{\log Q})_l$ with the 2, 4, 6, 8, 10, 12, and 14 largest deviations of $(\Delta\Delta \overline{\log Q})_n$ from the mean, and fit the rest of data points of $\Delta \overline{\log Q}$ in Eq (32):

$$\Delta \overline{\log Q} \approx d(\overline{\log Q})\Delta t \approx dy = ax + b$$

Step 9. *Scenario 9* – Fit the 10 data points of $(\Delta \overline{\log Q})_l$ from the last 15 days which have the minimum deviations of $(\Delta\Delta \overline{\log Q})_n$ from the mean in Eq (31):

$$\Delta \overline{\log Q} \approx d(\overline{\log Q})\Delta t \approx dy = a$$

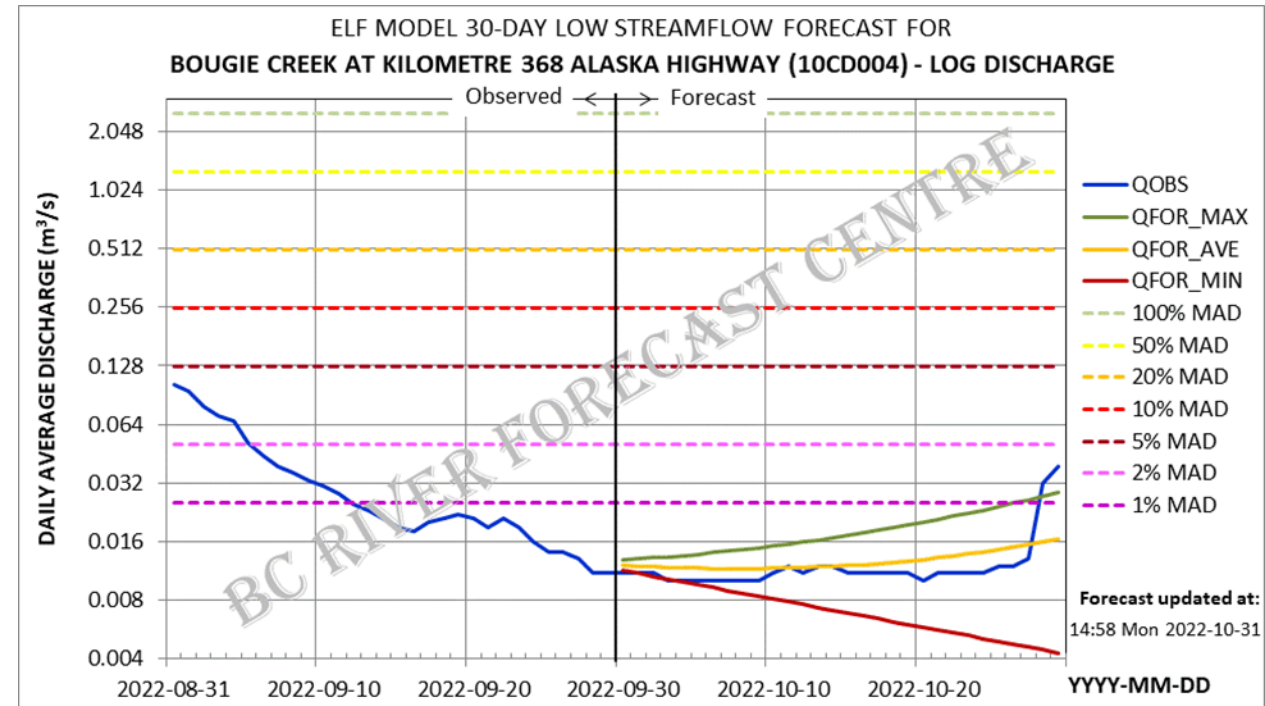
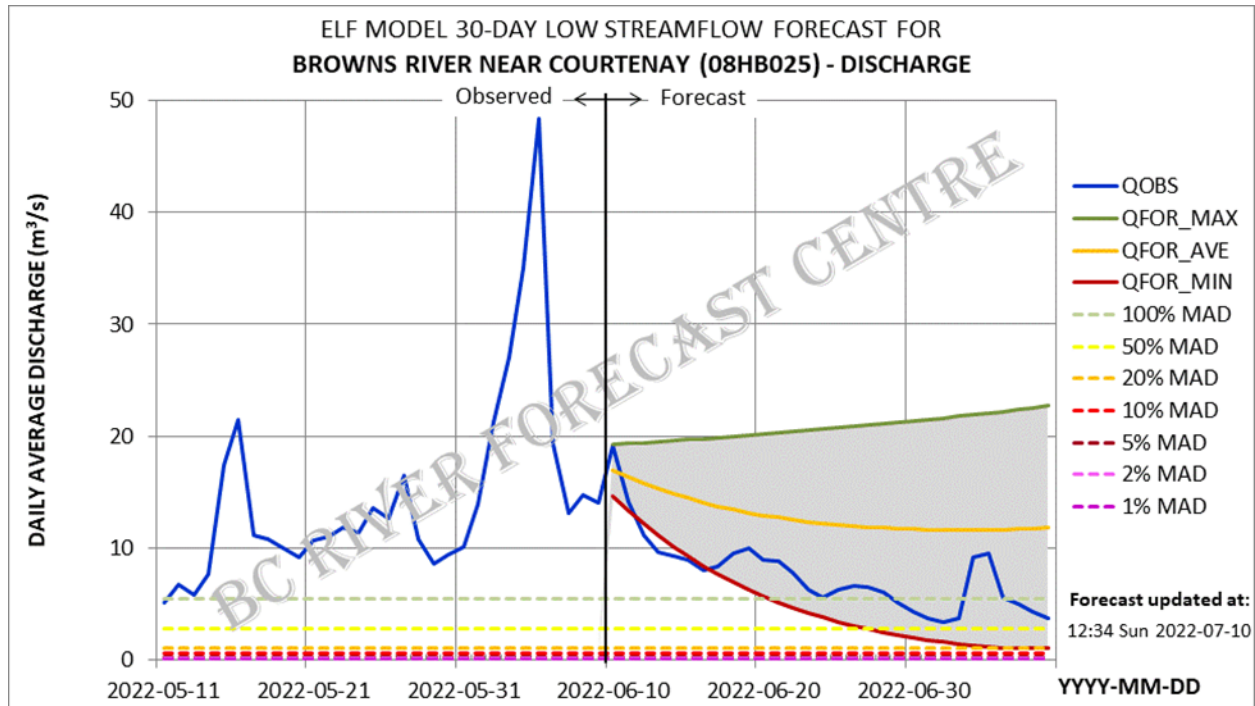
Or Eq (32) with: $\begin{cases} a = 0 \\ b = \bar{y} \end{cases}$ Eq (40)

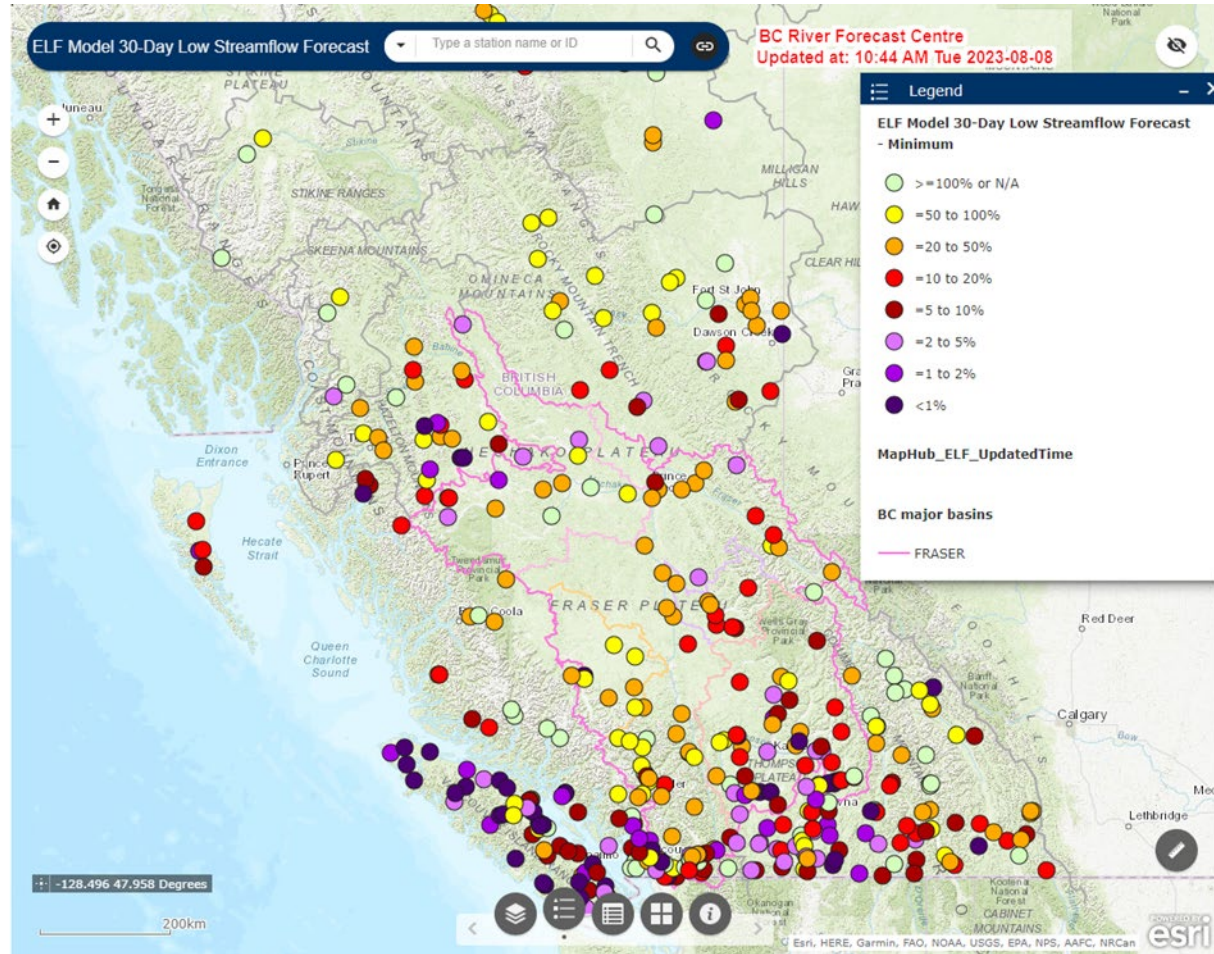
Step 10. *Scenario 10 to 12* – Fit the last 10, 5, and 2 data points of $(\Delta \overline{\log Q})_l$ in Equation (31), or Equation (32) with Equation (40), to estimate a and b .

Step 11. Finding the forecast maximum and minimum from the 12 scenarios for each day of the 30-day forecasting period, and the forecast average is the average of the forecast maximum and minimum.

Step 12. Restrict the forecast maximum and minimum and recalculate the forecast average when there was a recent rainfall/melt event.

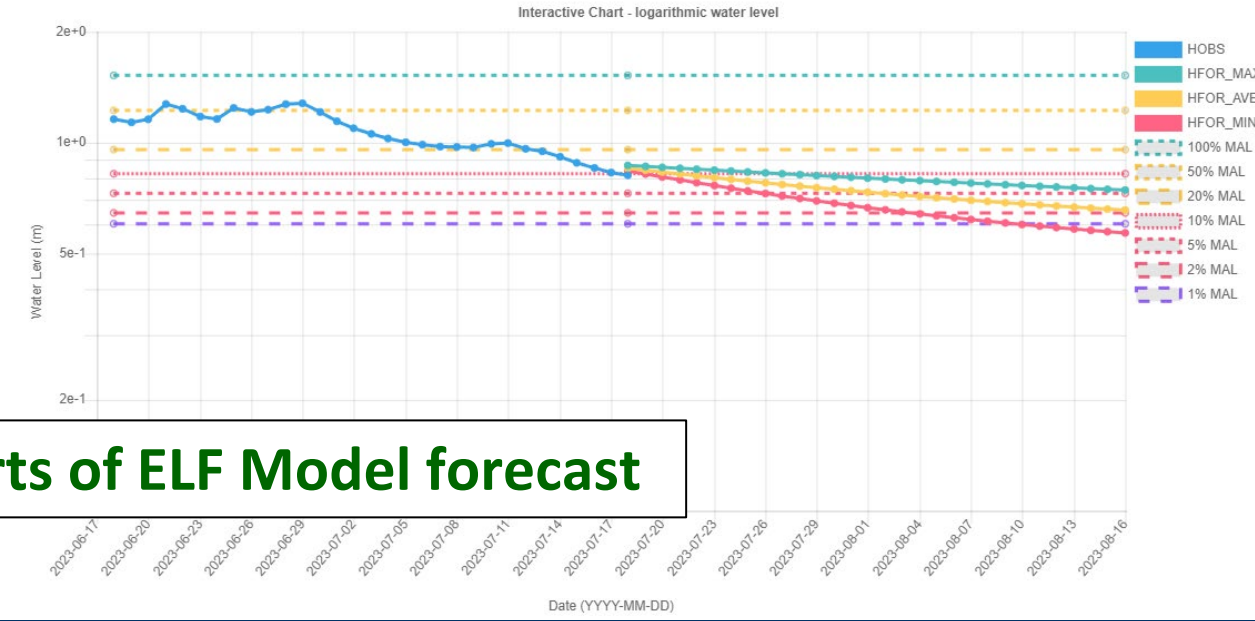
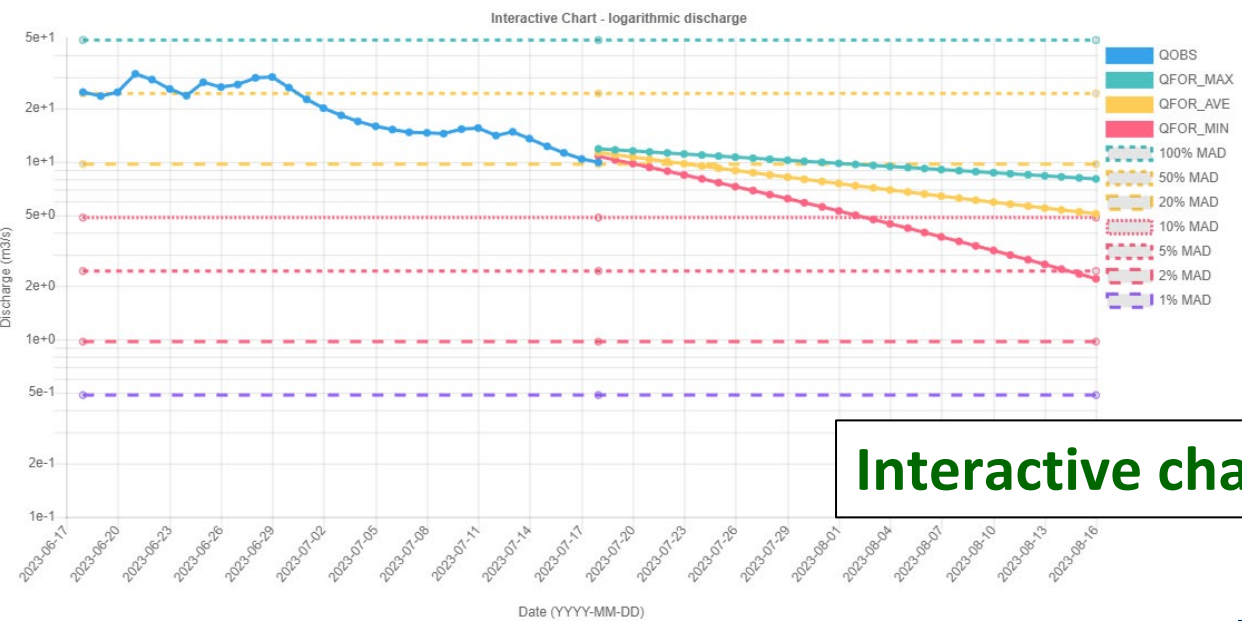
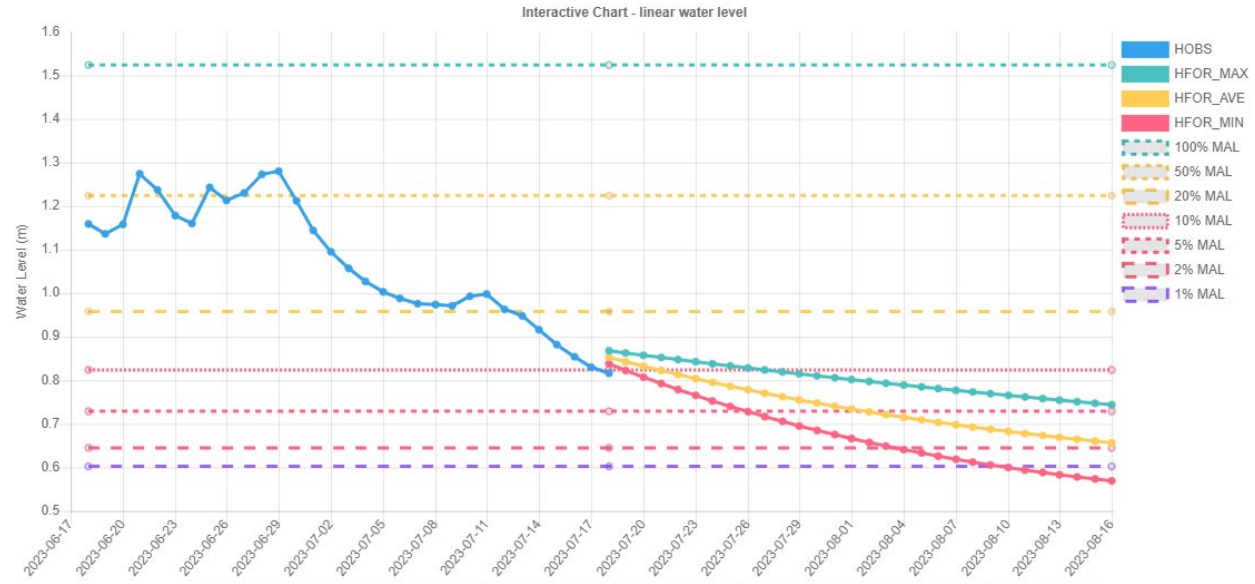
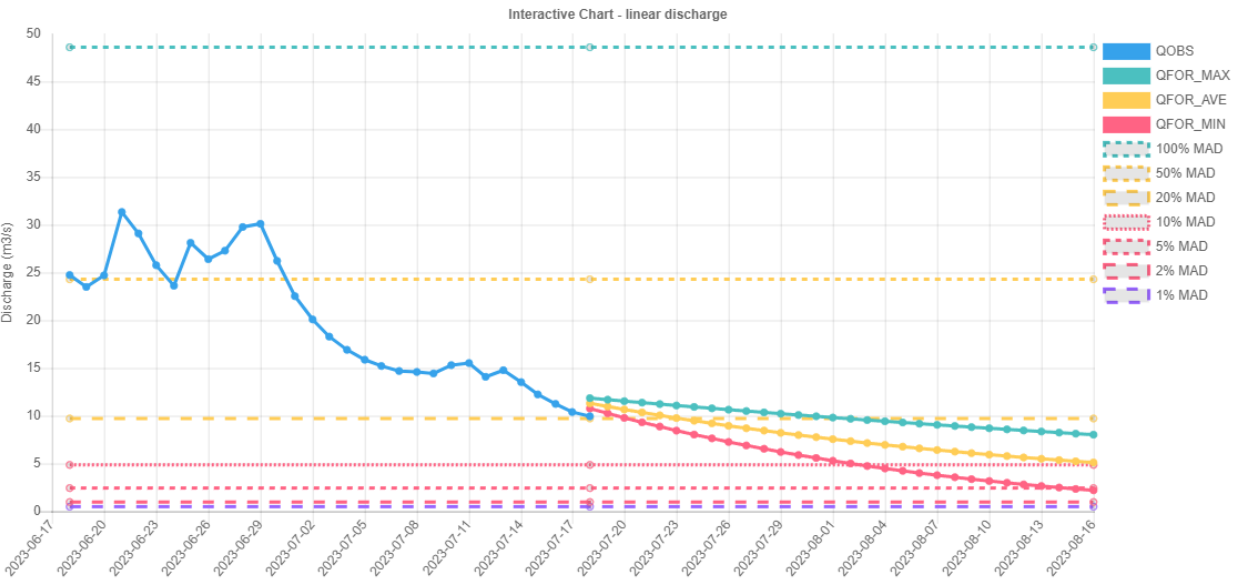
7. Data issues and twelve-step and twelve-scenario scheme



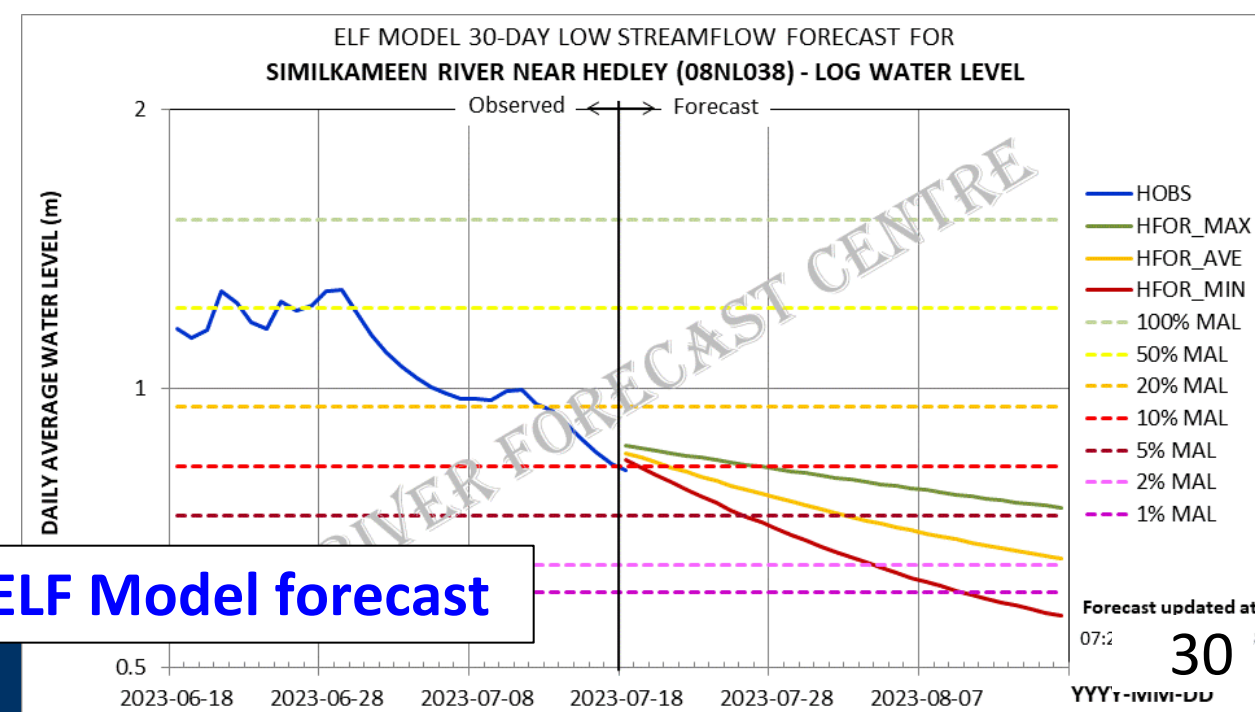
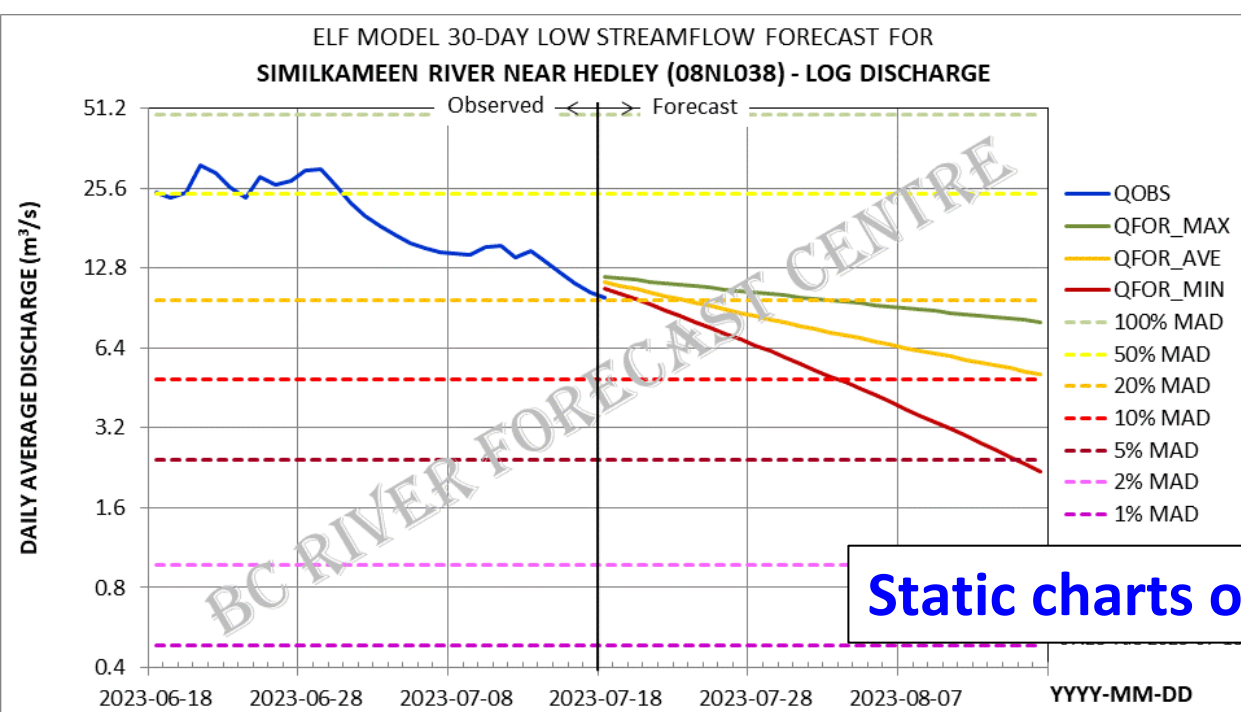
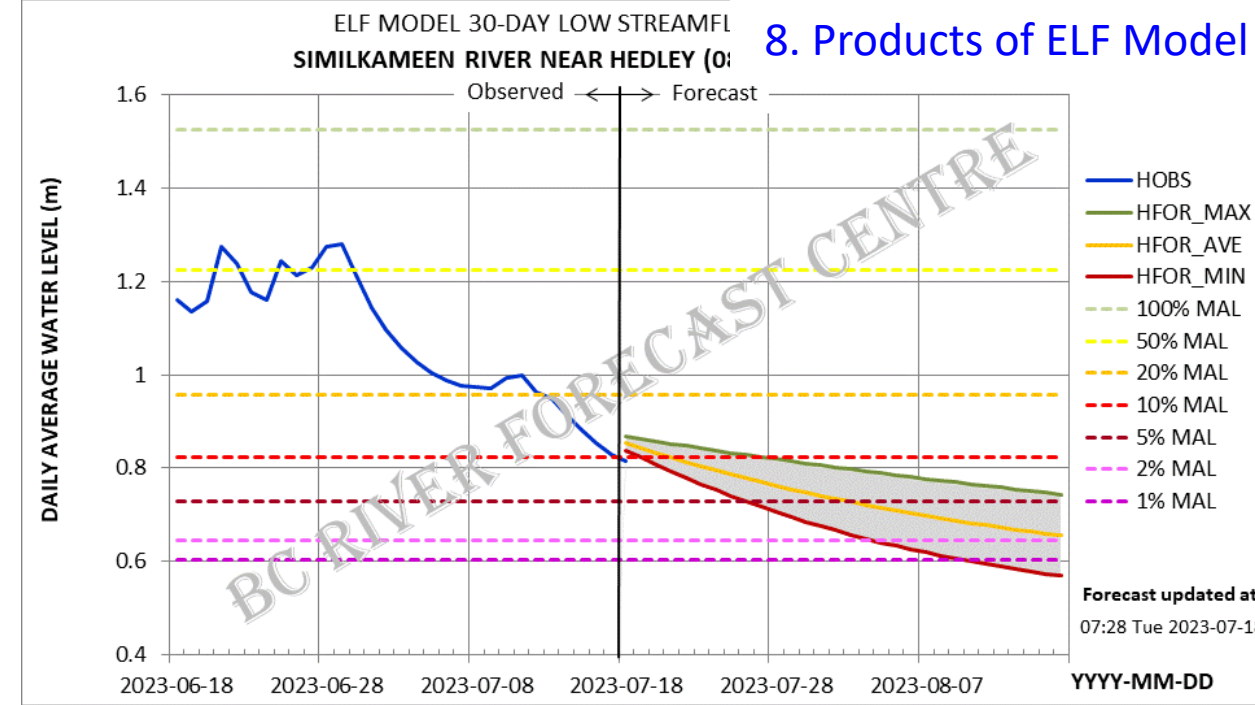
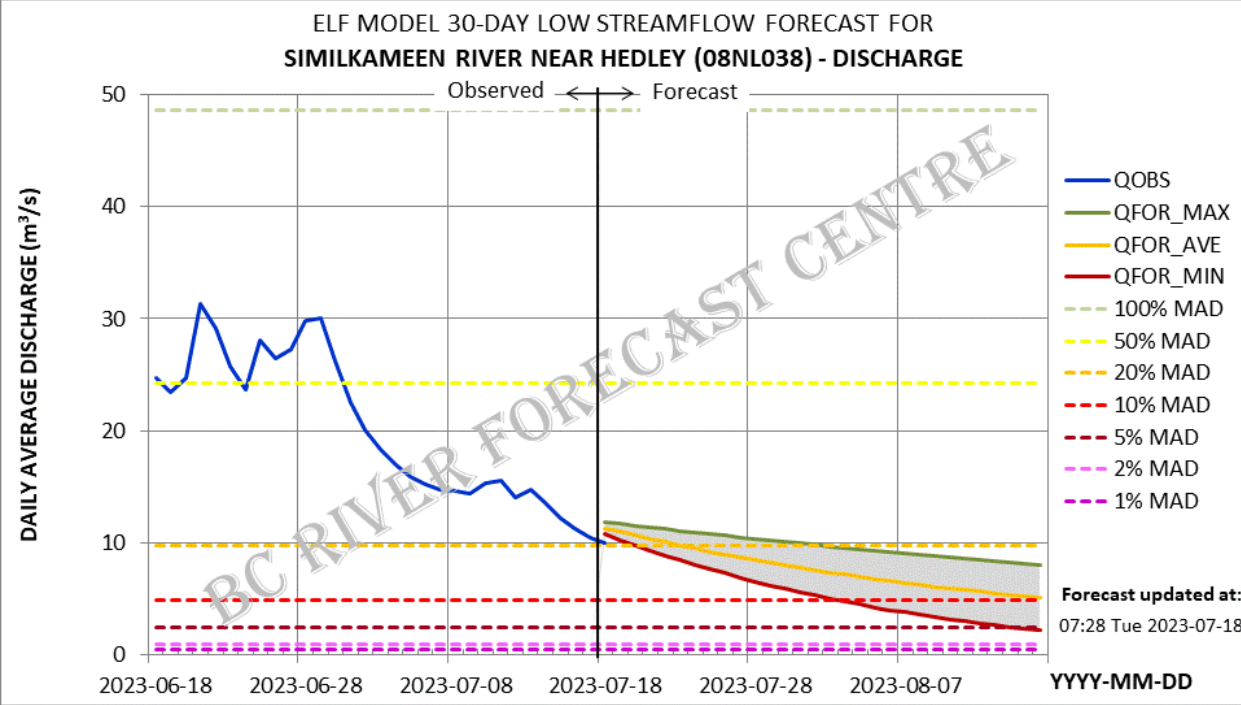


Maphub GIS map of ELF Model with color coded markers (http://bcrfc.env.gov.bc.ca/lowflow/map_elf.html)

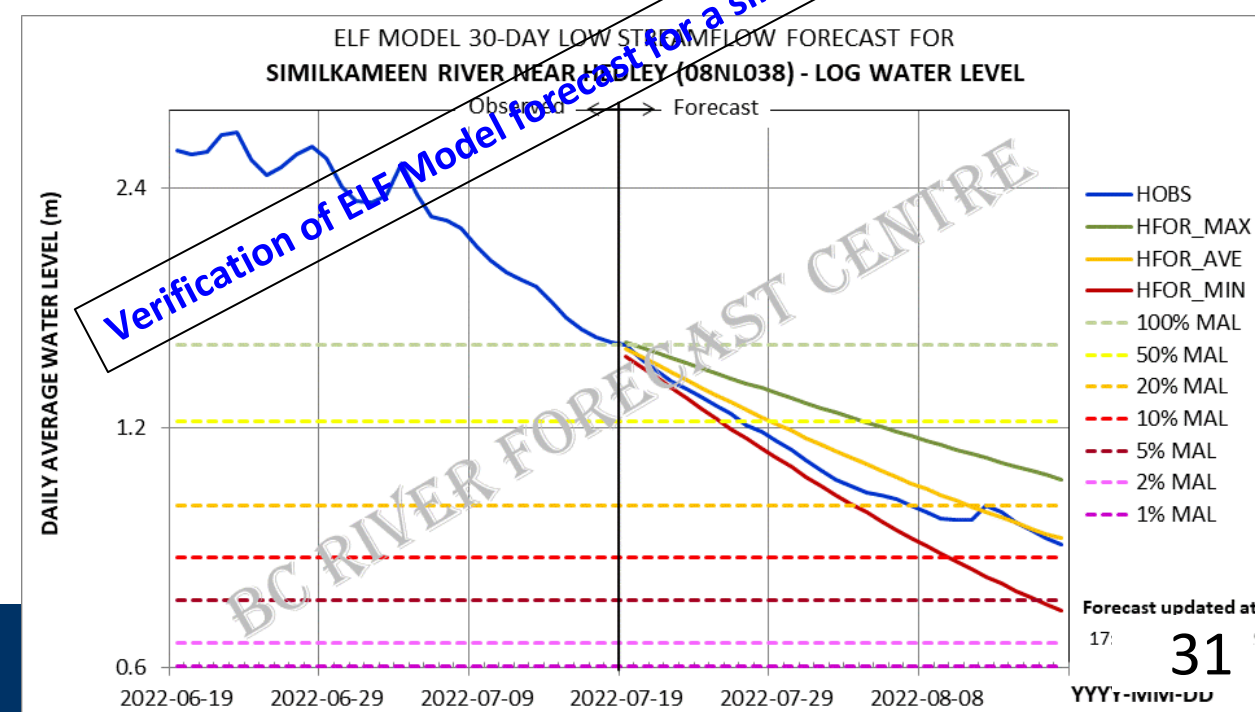
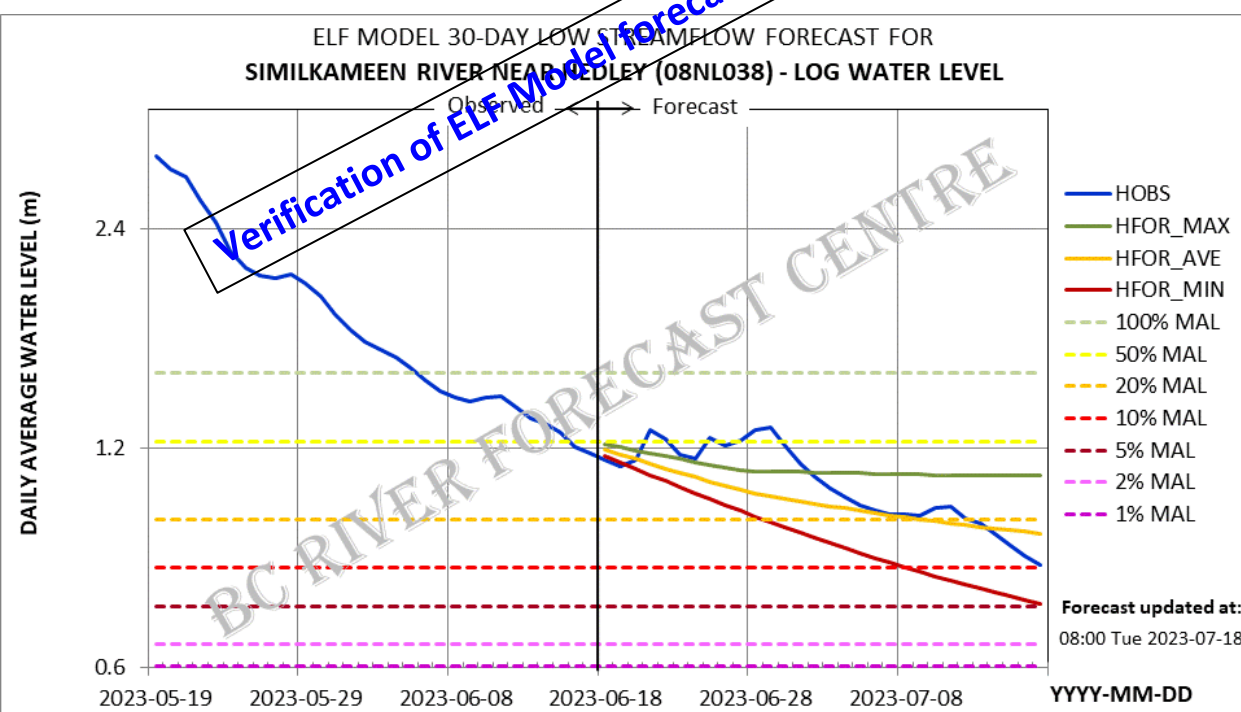
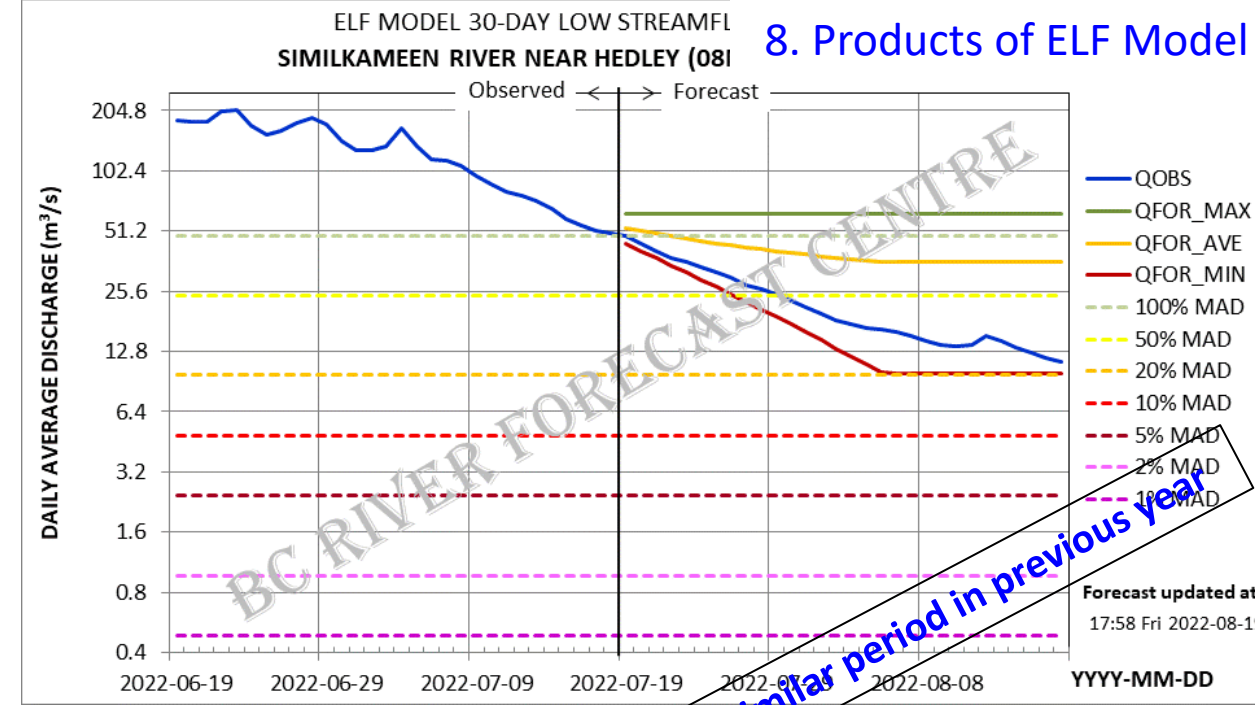
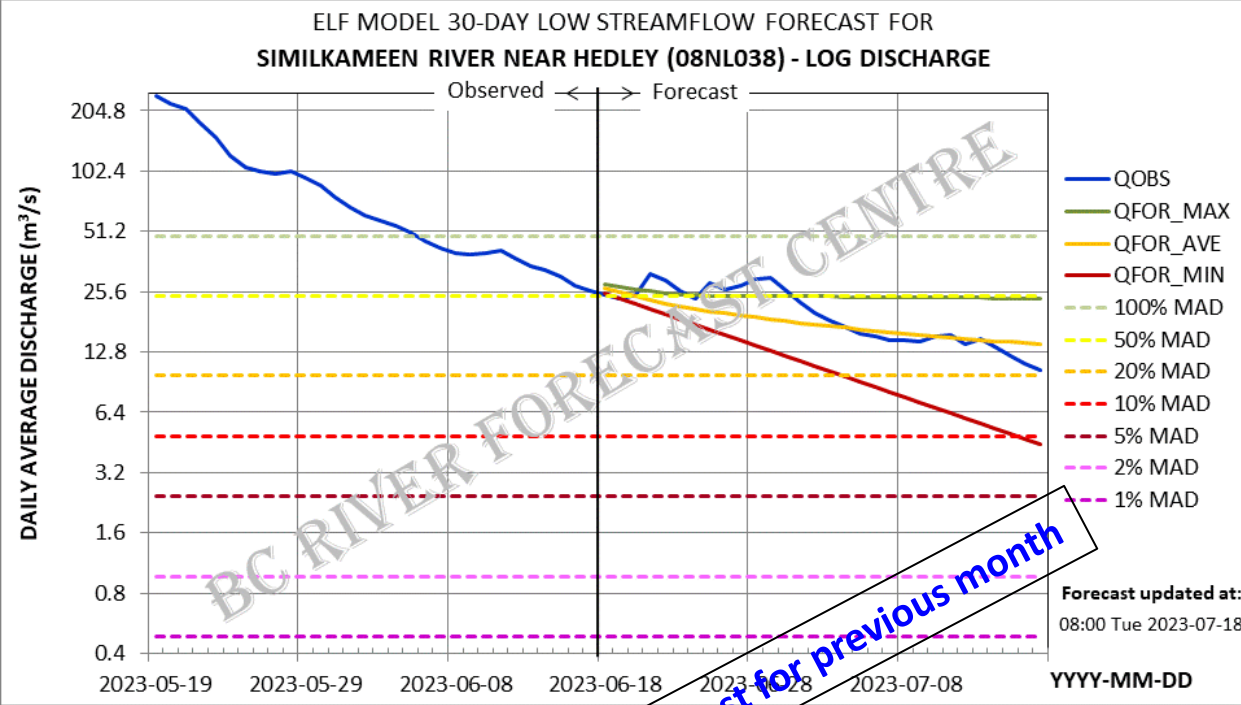




Interactive charts of ELF Model forecast



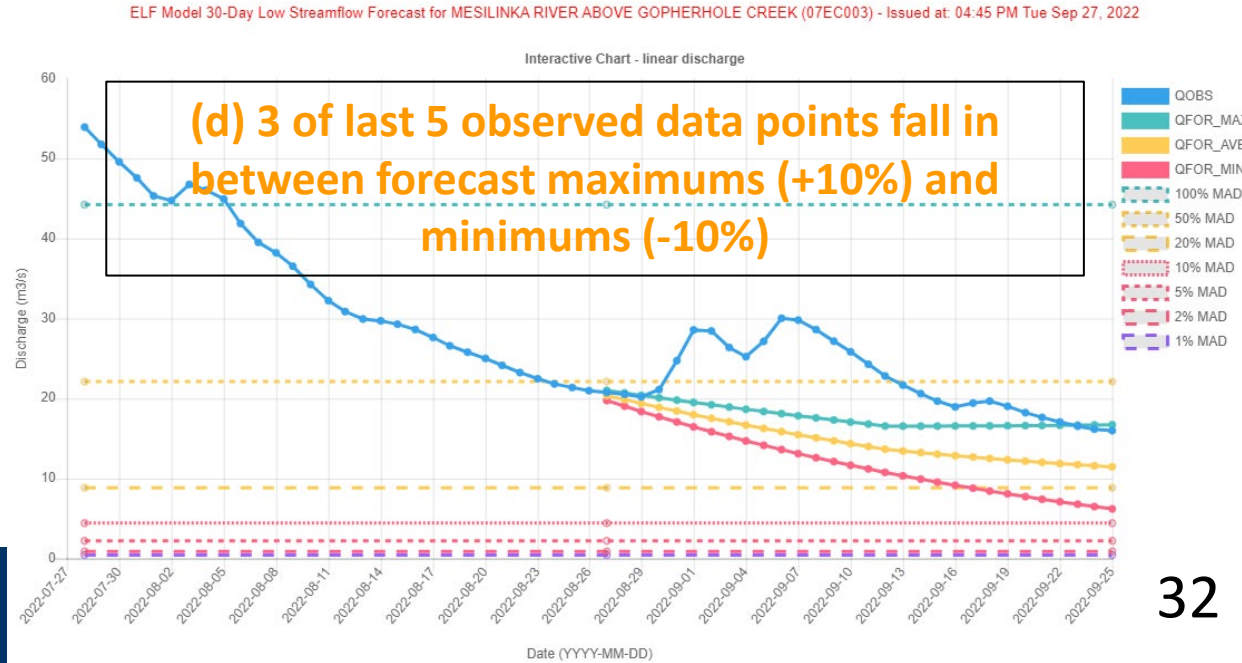
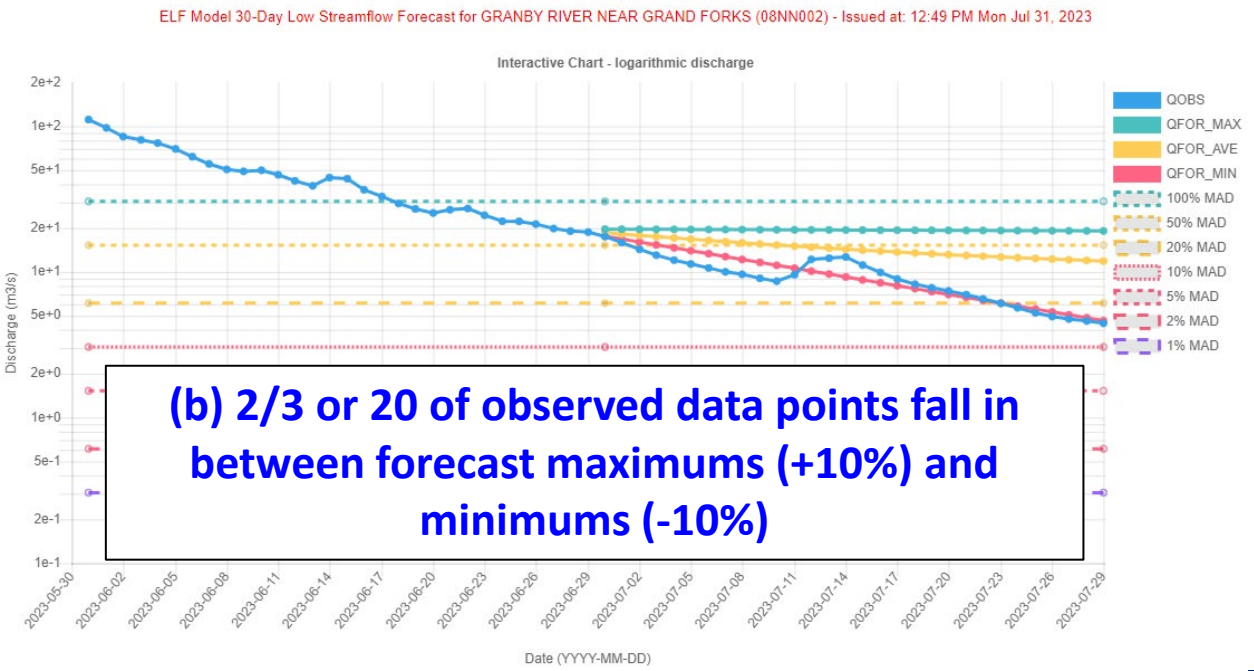
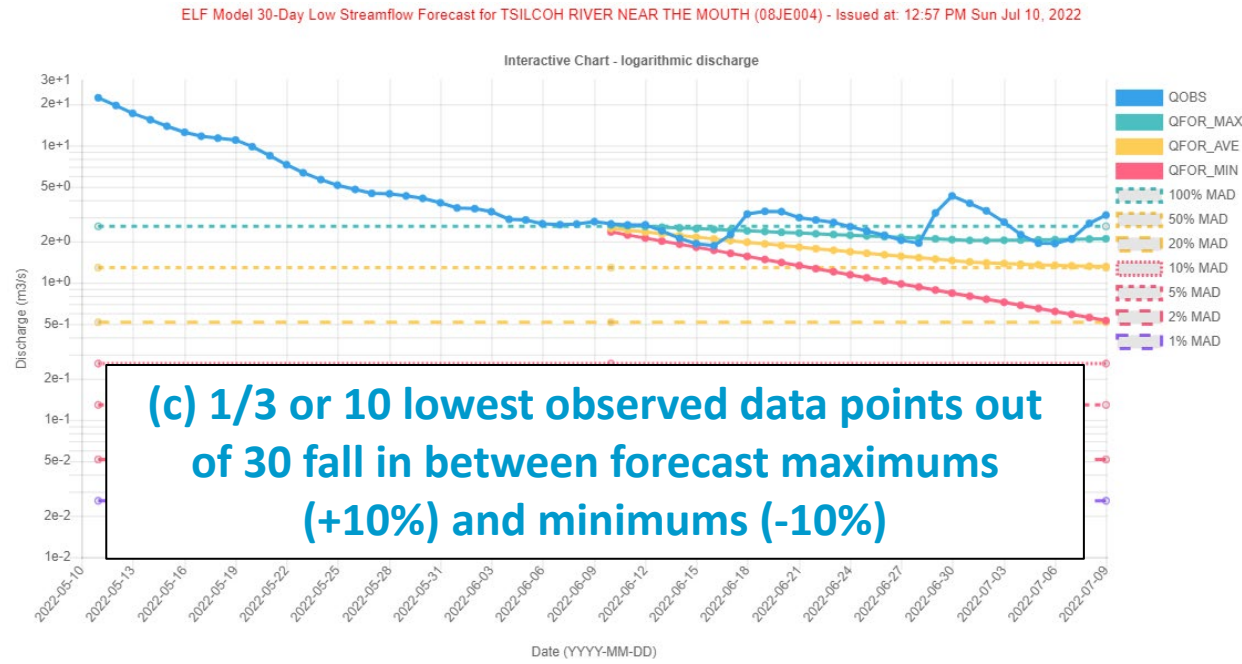
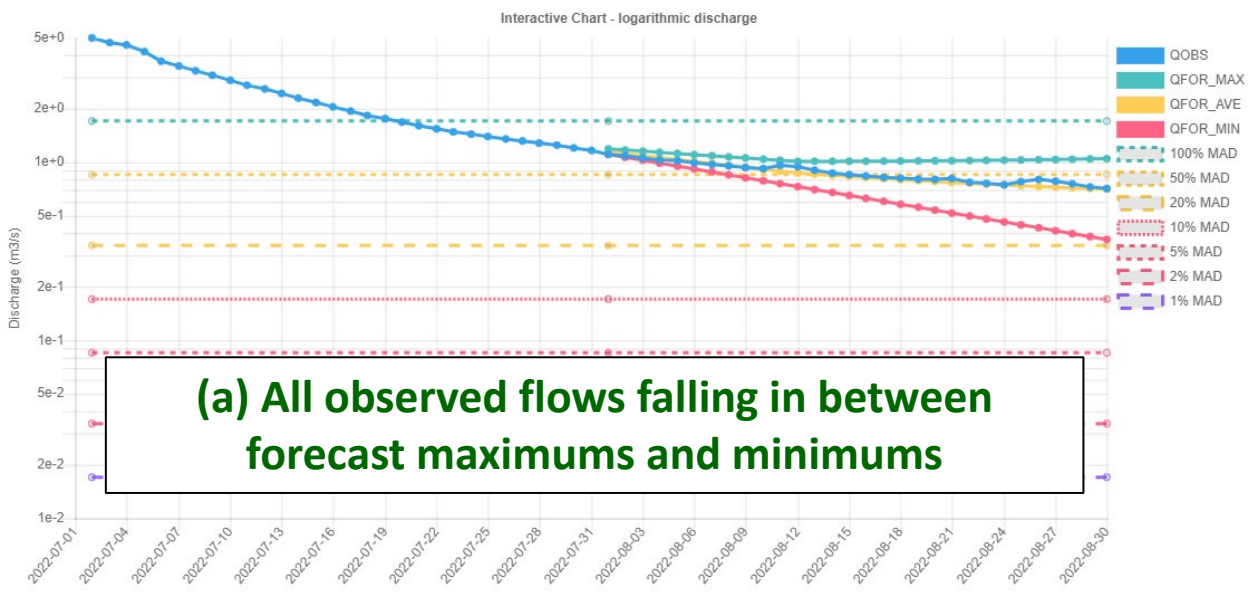
Static charts of ELF Model forecast



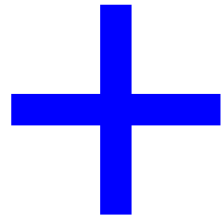
Verification of ELF Model forecast for previous month

Verification of ELF Model forecast for a similar period in previous year

9. Evaluation of ELF Model forecast accuracy



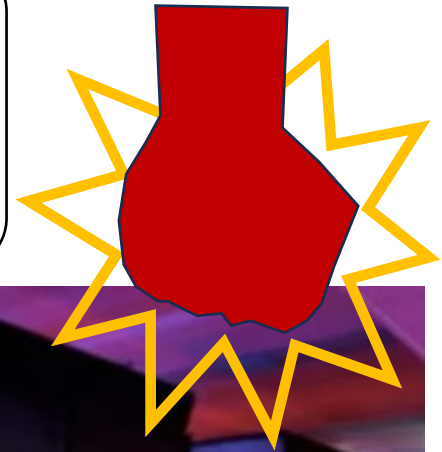
Reconstructed forecasts from January 2015 to June 2018



ELF Model started operational forecasting as July 2018



Eight (8) years of forecasts (800) for about 440 stations: $800 \times 440 = 352,000$ files of forecasts to open and read for a statistical analysis

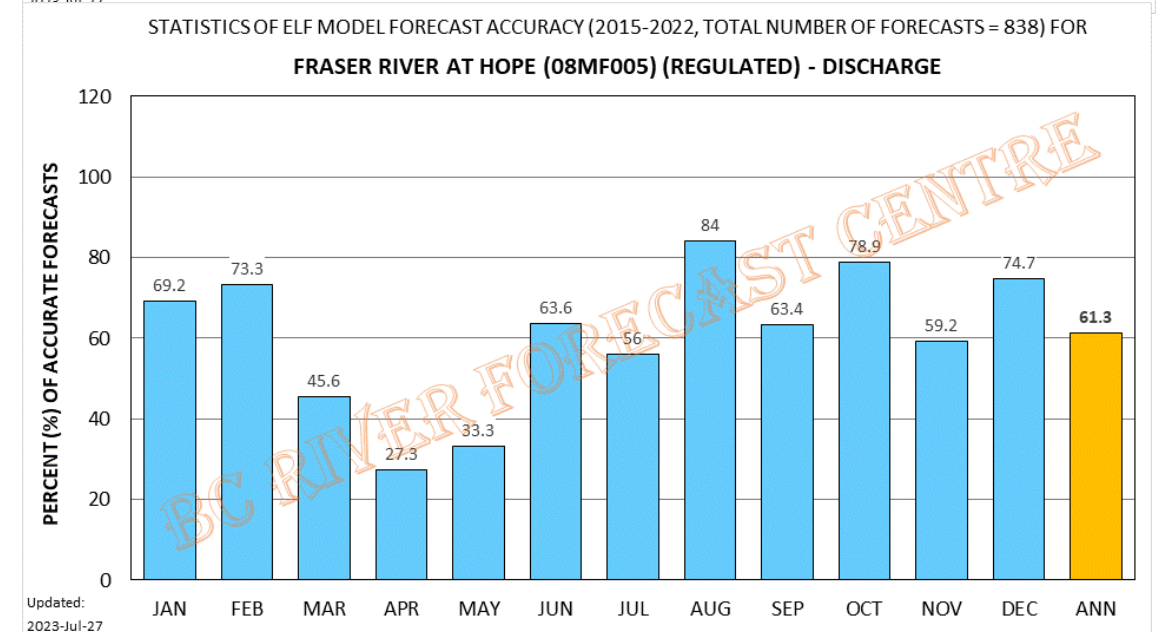
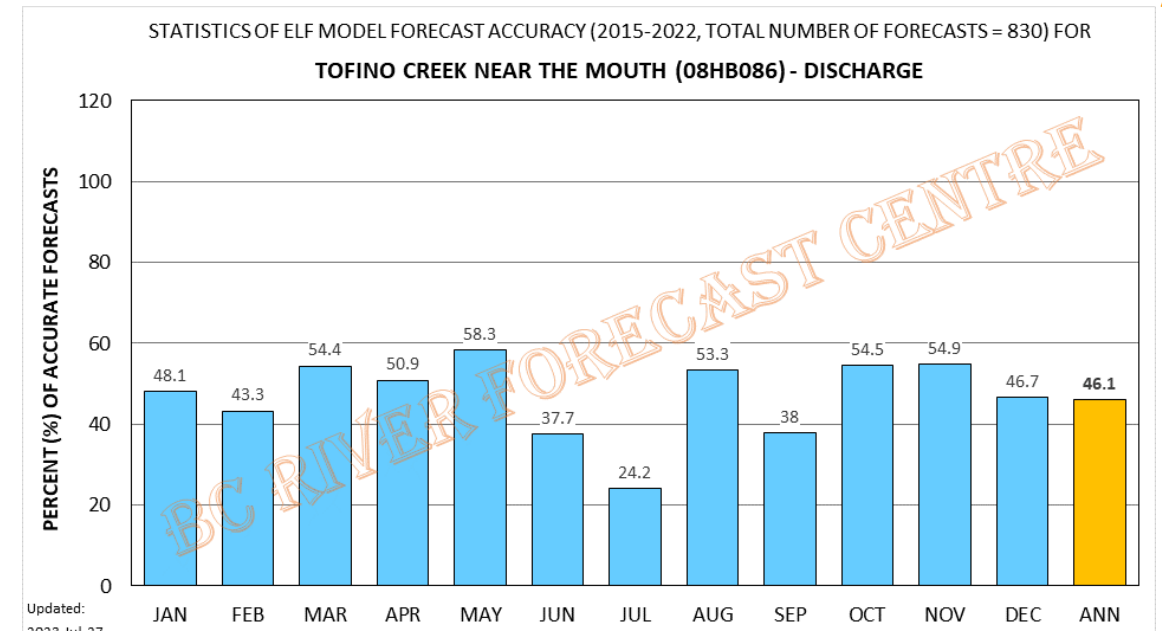


Making Kamloops GIS server "out of memories"!!!

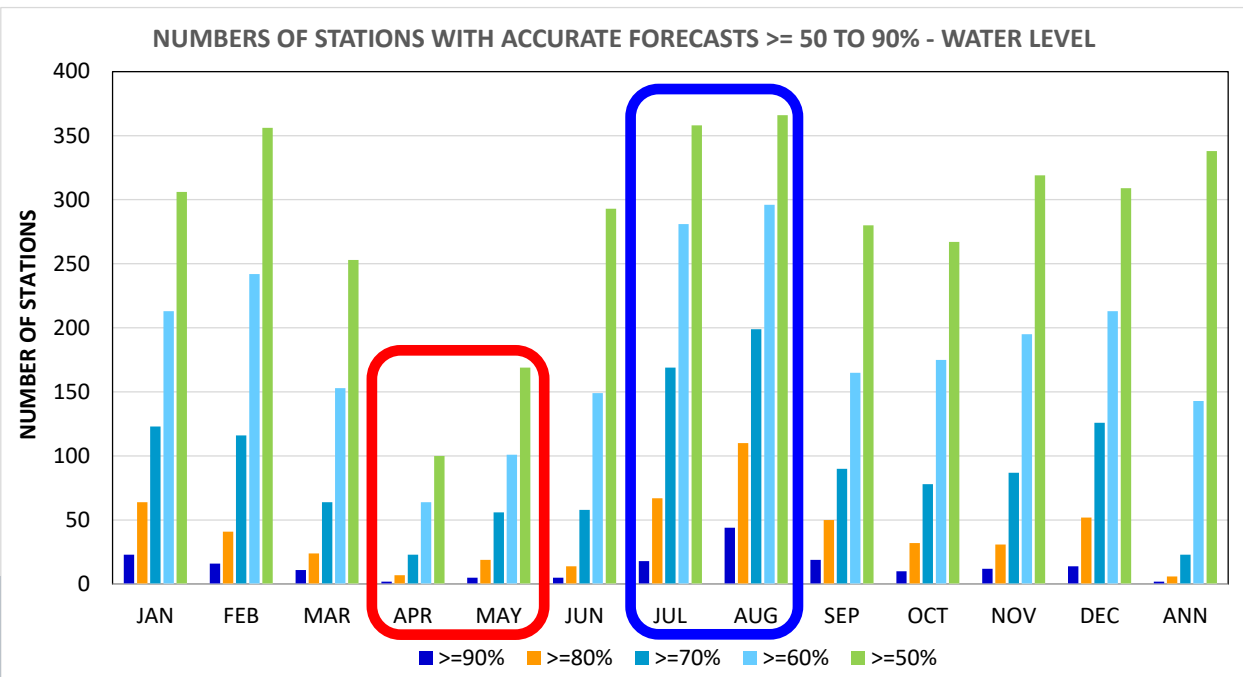
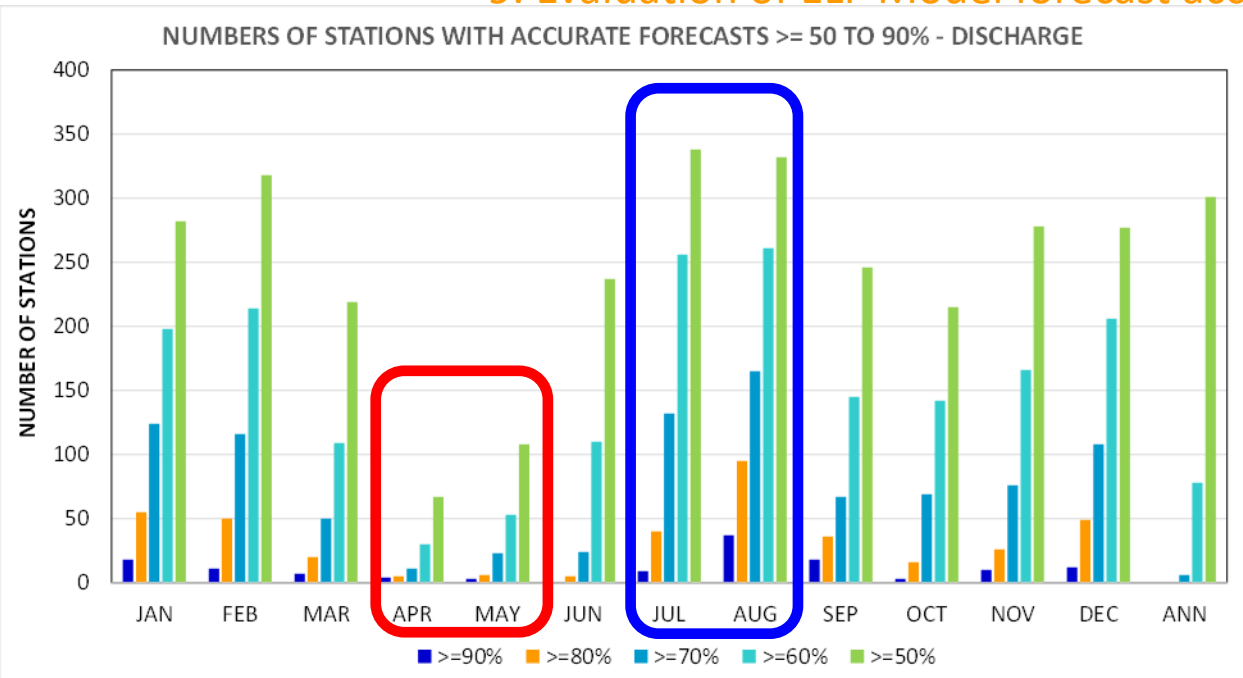
Consuming 40 hours of computing time of this server after program optimized.

9. Evaluation of ELF Model forecast accuracy

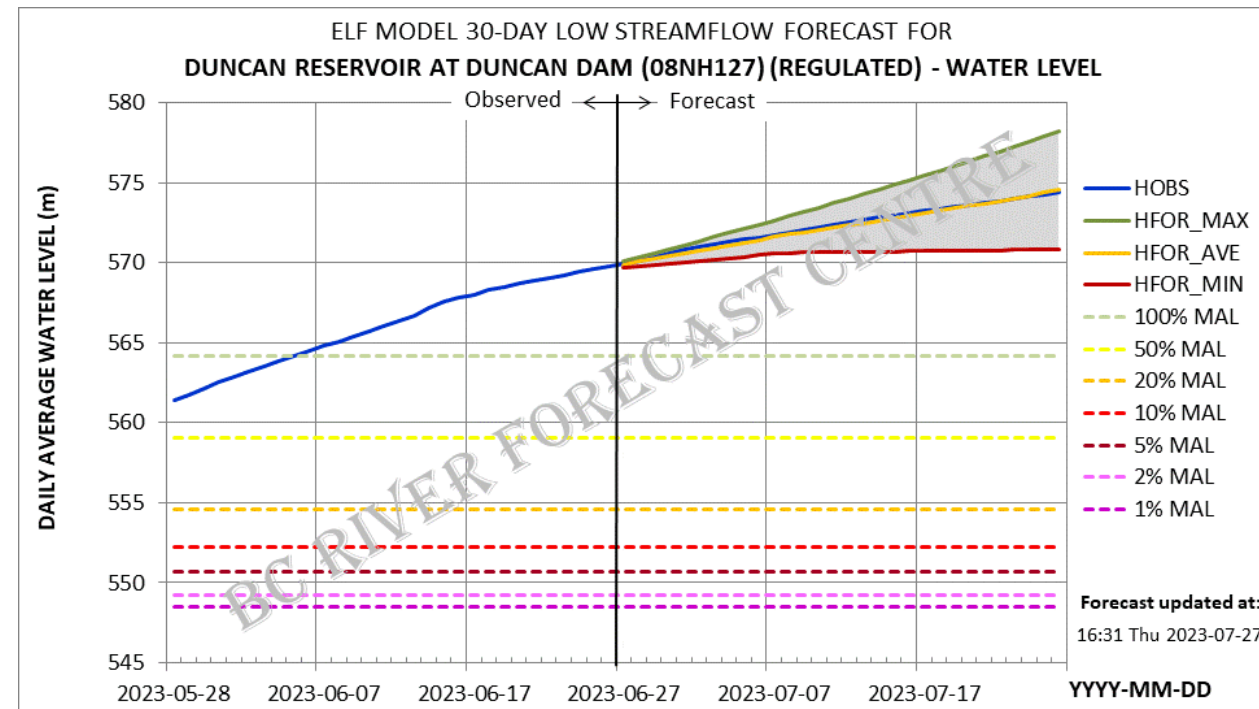
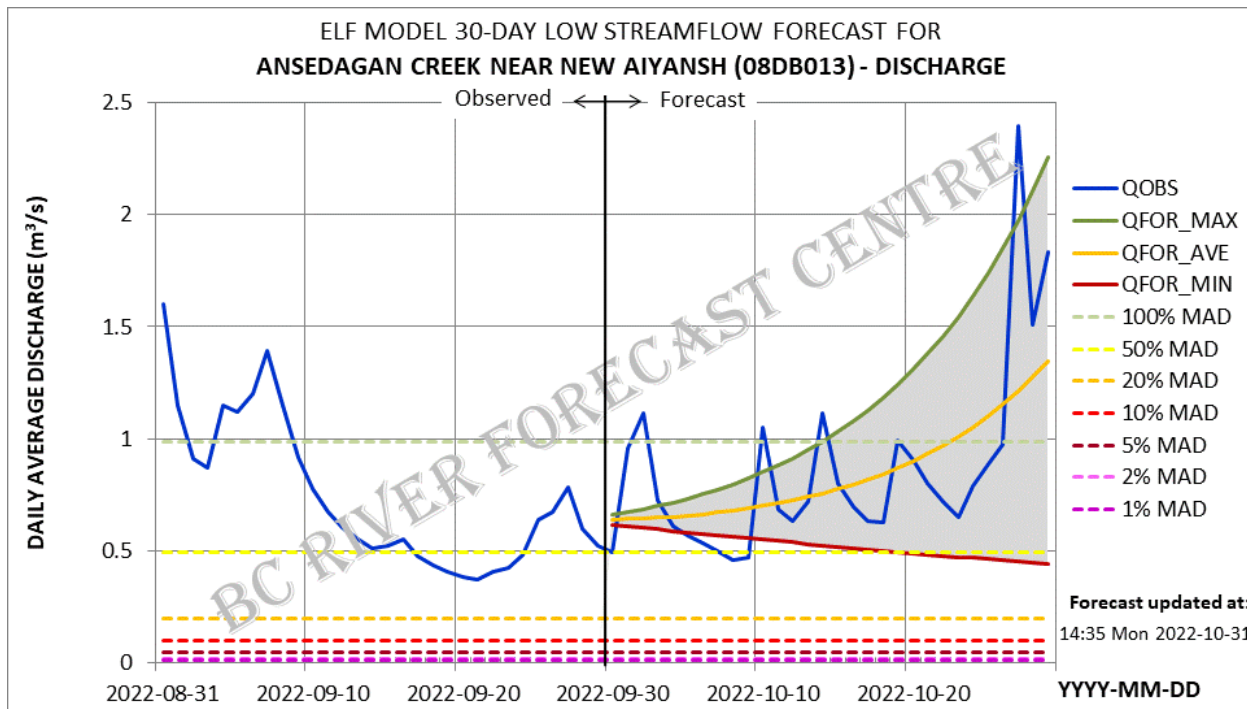
STATION ID	STATION NAME	PERCENT (%) OF ELF MODEL ACCURATE FORECASTS FOR DISCHARGES													TTL NO OF FOR
		JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	ANN	
08HB086	TOFINO CREEK NEAR THE MOUTH	48.1	43.3	54.4	50.9	58.3	37.7	24.2	53.3	38.0	54.5	54.9	46.7	46.1	830
08GB013	CLOWHOM RIVER NEAR CLOWHOM LAKE	48.1	60.0	49.1	29.1	40.3	64.9	57.1	49.3	53.7	46.5	57.7	76.0	53.5	838
08MH147	STAVE RIVER ABOVE STAVE LAKE	55.8	55.0	54.4	25.5	38.9	57.1	59.3	53.3	57.3	59.2	63.4	76.0	55.4	838
08GF007	WAKEMAN RIVER BELOW ATWAYKELLESE RIVER	53.8	63.3	49.1	36.4	38.9	68.8	69.2	62.7	58.5	35.2	56.3	69.3	56.1	838
08GA071	ELAHO RIVER NEAR THE MOUTH	65.4	56.7	28.1	14.5	30.6	54.5	61.5	41.3	57.3	64.8	76.1	78.7	53.6	838
08GE002	KLINAKLINI RIVER EAST CHANNEL (MAIN) NEAR THE MOUTH	63.5	73.3	49.1	10.9	27.8	33.8	71.4	61.3	52.4	73.2	69.0	61.3	54.7	838
08CG001	ISKUT RIVER BELOW JOHNSON RIVER	51.9	50.9	70.6	14.8	15.3	61.0	58.2	60.0	65.9	64.8	57.7	54.7	52.9	824
08DB001	NASS RIVER ABOVE SHUMAL CREEK	75.0	63.3	52.6	10.9	30.6	61.0	57.1	64.0	65.9	53.5	57.7	78.7	56.6	838
08CE001	STIKINE RIVER AT TELEGRAPH CREEK	59.6	66.7	86.0	12.2	15.3	54.5	65.9	37.3	52.4	50.7	63.4	36.0	50.2	832
08EF001	SKEENA RIVER AT USK	61.5	66.7	52.6	9.1	29.2	55.8	75.8	62.7	59.8	64.8	69.0	56.0	56.4	838
08NJ026	DUHAMEL CREEK ABOVE DIVERSIONS	71.2	61.7	22.8	9.1	25.0	36.4	70.3	82.7	63.4	53.5	52.1	74.7	53.3	838
08NG077	ST. MARY RIVER BELOW MORRIS CREEK	82.1	54.5	54.5	16.7	46.7	35.9	76.1	86.0	34.5	69.6	59.6	51.0	56.0	493
08NF001	KOOTENAY RIVER AT KOOTENAY CROSSING	73.1	60.0	60.0	14.5	33.3	51.9	68.1	76.0	62.2	78.9	67.6	58.7	59.4	836
08NG002	BULL RIVER NEAR WARDNER	67.3	70.0	24.6	12.7	51.4	46.8	76.9	93.3	70.7	67.6	67.6	82.7	62.9	838
08NN026	KETTLE RIVER NEAR WESTBRIDGE	75.0	60.0	20.0	9.4	33.3	51.9	65.9	80.0	46.3	46.5	53.5	66.7	52.0	834
08NL038	SIMILKAMEEN RIVER NEAR HEDLEY	42.3	53.3	38.6	14.5	43.1	50.6	80.2	86.7	50.0	38.0	42.3	54.7	51.4	838
08NG065	KOOTENAY RIVER AT FORT STEELE	71.2	63.3	59.6	12.7	29.2	53.2	75.8	89.3	69.5	74.6	67.6	64.0	62.1	838
08LF051	THOMPSON RIVER NEAR SPENCES BRIDGE	78.8	82.1	64.9	18.2	37.5	59.7	52.7	88.0	61.0	76.1	70.4	88.0	64.9	834
08MC018	FRASER RIVER NEAR MARGUERITE	71.2	51.7	59.3	28.3	47.2	59.7	68.1	80.0	62.2	59.2	62.0	78.7	61.6	833
08MF005	FRASER RIVER AT HOPE	69.2	73.3	45.6	27.3	33.3	63.6	56.0	84.0	63.4	78.9	59.2	74.7	61.3	838



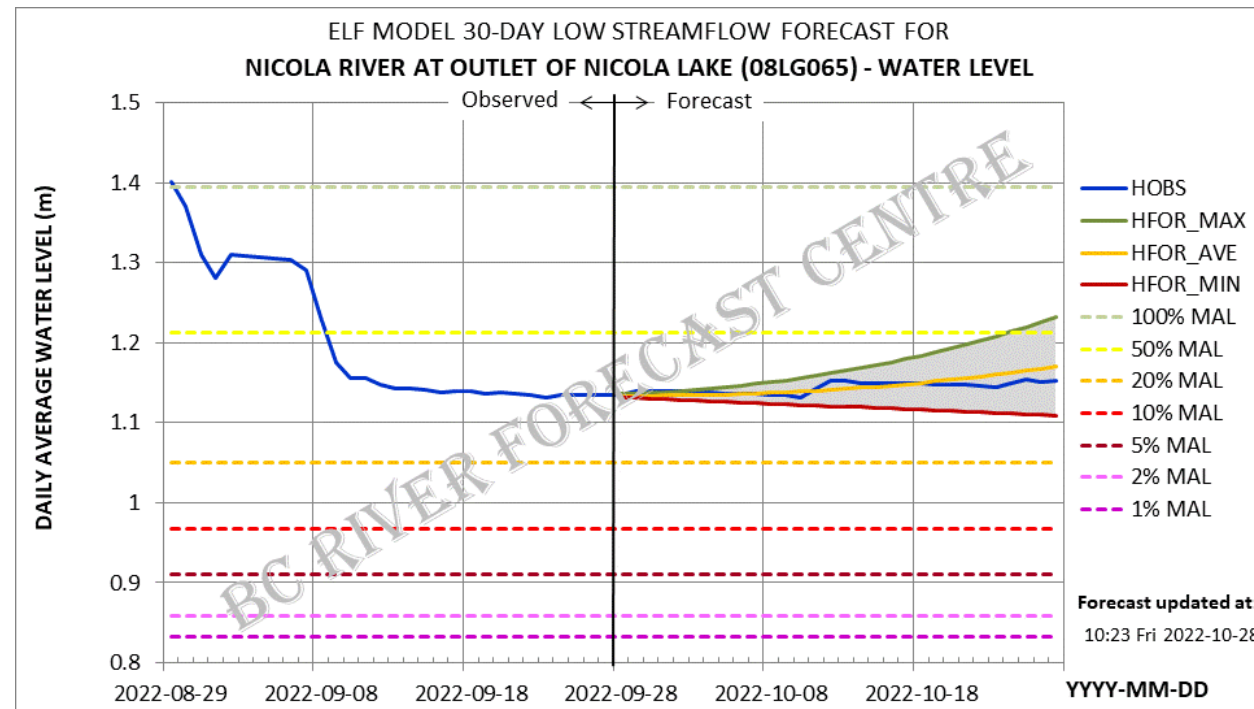
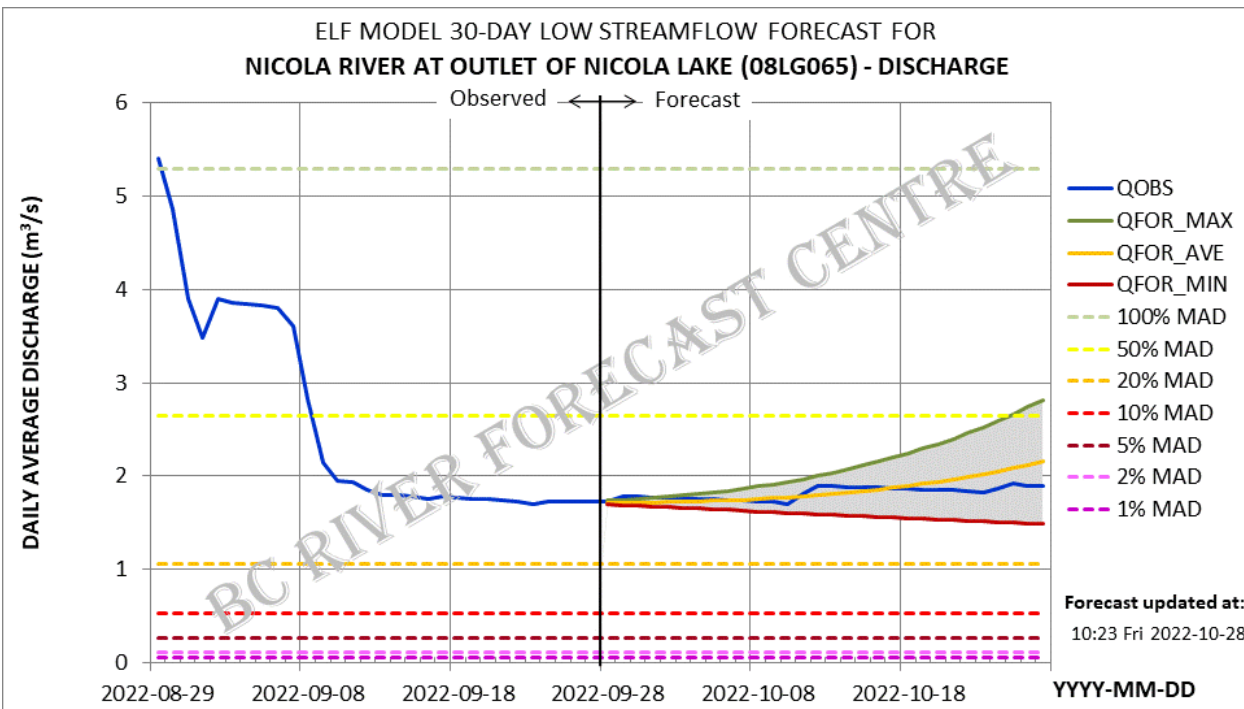
Month	NUMBERS OF STATIONS WITH ACCURATE FORECASTS >= 50 TO 90%									
	FORECASTS OF DISCHARGE					FORECASTS OF WATER LEVEL				
	>=90%	>=80%	>=70%	>=60%	>=50%	>=90%	>=80%	>=70%	>=60%	>=50%
JAN	18	55	124	198	282	23	64	123	213	306
FEB	11	50	116	214	318	16	41	116	242	356
MAR	7	20	50	109	219	11	24	64	153	253
APR	4	5	11	30	67	2	7	23	64	100
MAY	3	6	23	53	108	5	19	56	101	169
JUN	0	5	24	110	237	5	14	58	149	293
JUL	9	40	132	256	338	18	67	169	281	358
AUG	37	95	165	261	332	44	110	199	296	366
SEP	18	36	67	145	246	19	50	90	165	280
OCT	3	16	69	142	215	10	32	78	175	267
NOV	10	26	76	166	278	12	31	87	195	319
DEC	12	49	108	206	277	14	52	126	213	309
ANN	0	0	6	78	301	2	6	23	143	338



- A. July and August are the two months that there are the largest numbers of stations that the ELF Model has large percents (>=50 to 90%) of accurate forecasts.
- B. April and May are the two months that there are the least numbers of stations that the ELF Model has large percents (>=50 to 90%) of accurate forecasts.
- C. Other months are in between.

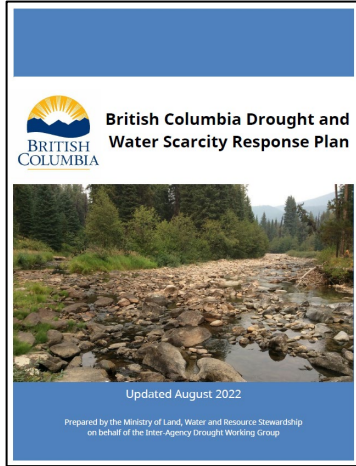


The ELF Model is run all year round using 30 day observed flow data (discharges and/or water levels) to produce forecasts for the next 30 days regardless the flow is rising or dropping. These forecasts of rise are **for information only** and are not recommended for management purposes because the ELF Model is not developed for forecasting rises and because the model **has no meteorological data input**.

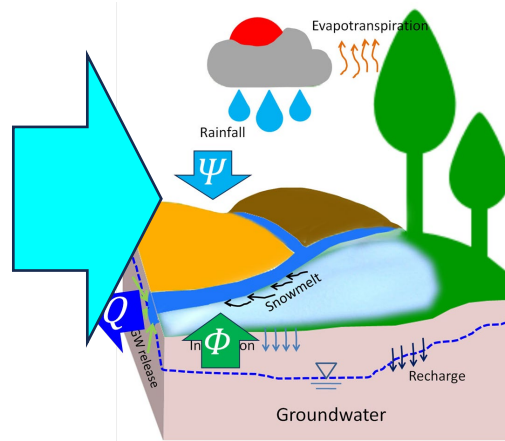


The regulated flow stations are not removed from the list of modeled stations. As such, the ELF Model also produce forecasts for the regulated stations and sometimes the forecasts are also accurate as long as the operation of the regulating facilities is consistent during the period of the model’s input data and the period of forecasting. These forecasts are also **for information only** and are not recommended for management purpose either because **the forecast accuracy is uncertain**.

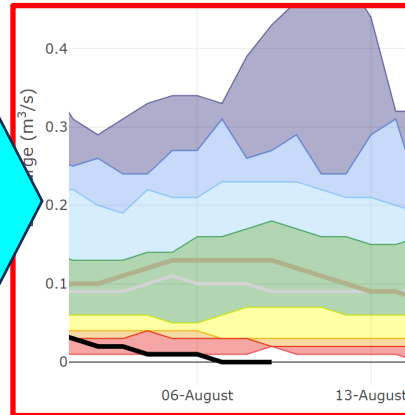
Background



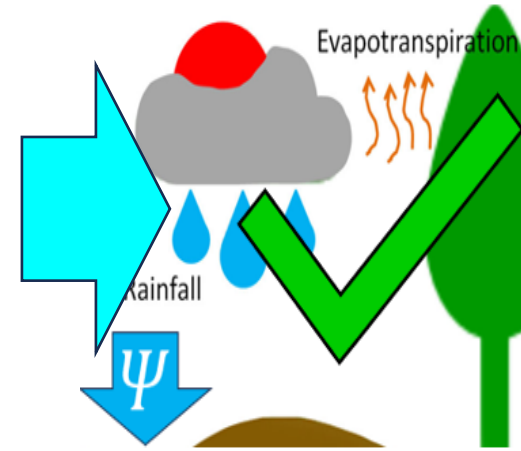
Definition of Low Flow



Why not hydro model



Fundamental assumption



Basic equations

$$Q(t) = Q_0 e^{-at}$$

$$\ln(Q) = -at + \ln(Q_0)$$



If 30 data, overdetermined: $f = y = ax + b$

$$R^2 = \sum_{i=1}^n [y_i - f(x_i, a_1, a_2, \dots, a_j, \dots, a_m)]^2$$

$$\frac{\partial(R^2)}{\partial a_i} = 0, i = 1 \text{ to } n$$

$$a = \frac{\sum_{i=1}^n (x_i y_i) - n \bar{x} \bar{y}}{\sum_{i=1}^n x_i^2 - n \bar{x}^2}$$

$$b = \frac{\bar{y} \sum_{i=1}^n x_i^2 - \bar{x} \sum_{i=1}^n (x_i y_i)}{\sum_{i=1}^n x_i^2 - n \bar{x}^2}$$

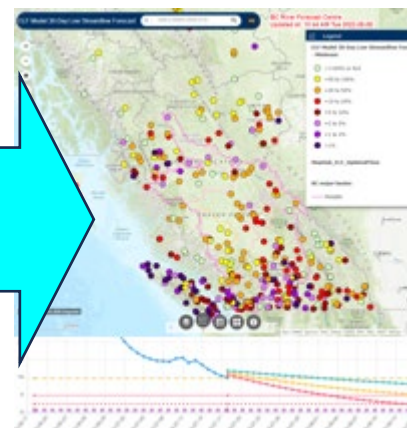
$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

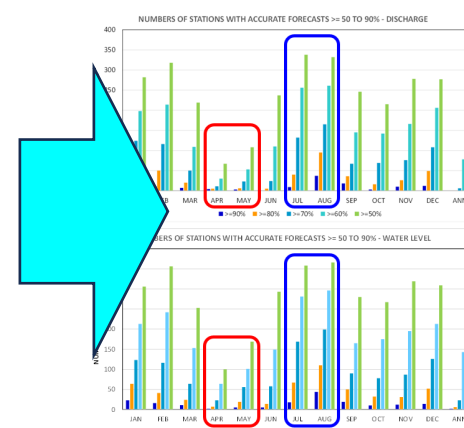
Solve overdetermined system exp res equ.



Data issues & 12-step /12 scenario scheme



Products of ELF Model



Evaluation of ELF M forecast accuracy

Conclusion:
The ELF Model can produce accurate low flow forecast when streamflow conditions fulfill the fundamental assumption and the basic equation.

THANK YOU!

QUESTIONS?



Charles.Luo@gov.bc.ca

The Extrapolating Logarithmic Flow (ELF) Model